# Expenditure rules in the context of a balanced budget

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**Abstract**

Expenditure rules that are one of the types of fiscal rules are important in budgetary policies. They facilitate to maintain a stable budget in accordance with the adopted strategy in the medium and long term, they allow to coordinate the budgetary expenditure and they enable mitigate the effects of negative shocks associated with budgetary revenue. The aim of the article is the presentation of the method of determination of expenditure rules based on the optimal control model in the context of a balanced budget. In this paper we present the fiscal policy rule that is the solution of the Quadratic Linear Problem. This rule help develops the economy in accordance with the desired path and this rule minimize the deviation of inflation rate, GDP growth and unemployment rate from the desired values

***Keywords:*** *expenditure rule, balanced budget, economic policy model, control theory*

***JEL Classification:*** E62, C54, C61, H50

## 1 Introduction

Government actions that affect on the country's economic situation and its development are taken through the implementation of economic policy. The monetary and fiscal policies are important in the realization of economic policy in the short and medium term. Government making decisions about the fiscal policy may affect on the realization of sustainable economic development. Often used method of fiscal decision-making are decisions based on the rules. In this paper we determine the optimal fiscal policy rules based on optimal control theory model. This optimal fiscal rules are a solution of the economic policy model consists of the function criterion and the model of the economy. The model of fiscal policy, which we use, should lead to long-term stable economic growth. Thus, models of economic policy can be the basis for determination of the strategy which the effect is the achievement of the desired values of selected variables, such as inflation, output and unemployment in the future.

The other authors analyze the optimal fiscal policy. The optimal fiscal policy in a stochastic endogenous growth model with private and public capital is studied by Tamai (2016). Public investment, the rate of return, and optimal fiscal policy in a stochastically growing economy are analyzed by Tamai (2016). The optimal fiscal rules within a stochastic

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model of Keynesian type are analyzed by Correani et al. (2014). About Taylor rule for fiscal policy write Kliem and Kriwoluzky (2014) and Kendrick and Amman (october 2011).

## 2 The importance of fiscal policy based on rules

The fiscal policy involves the government decisions on the size and structure of public expenditure, budget deficit and public debt. Fiscal sustainability, as an integral part of macroeconomic stability will strengthen the protection of the economy against various types of shocks. One of the ways of making decisions are the decisions based on rules. When we conduct the rules-based fiscal policy, it is strengthened the prudence and the objectivity in the realization of fiscal policy. The fiscal rules have a significant impact on the economy. One of the benefits is the creation of favorable conditions for increase of potential GDP growth. The decisions on the basis of the rules, including the expenditure rule, allow to coordinate the budgetary expenditure. These decisions allow to mitigate the effects of negative shocks related to budgetary revenue and they help maintain a stable budget in accordance with the adopted strategy in the medium and long term. If fiscal impulse is necessary for stimulation of economic business, the emergence of a budget deficit should be possible. However, it should be noted that the fiscal impulse cannot lead to a permanent increase of budget deficit and public debt.

Furthermore, it should also be noted that Article 5 of the Council Directive of the European Union (COUNCIL DIRECTIVE 2011/85/EU of 8 November 2011 on requirements for budgetary frameworks of the Member States) says that „Each Member State shall have in place numerical fiscal rules which are specific to it and which effectively promote compliance with its obligations deriving from the TFEU (ed. of the Treaty on the functioning of the European Union) in the area of budgetary policy over a multiannual horizon for the general government as a whole…" Whereas, the Article 7 of this Directive requires that „the annual budget legislation of the Member States shall reflect their country-specific numerical fiscal rules in force".

Currently in Poland it holds true the modification of stabilizing expenditure rule. Therefore, the basis for the analysis work is to determine the expenditure rule in the context of a balanced budget. The application of these rules allows the economy to develop according to the desired path.

In order to determine these rules, we applied the control theory and the selected form of dynamic model of fiscal and monetary policies.

## 3 Economic Policy Model

A typical economic policy model (EPM) in deterministic version consists of optimized function criterion (FC) and the model describing the economy (EM), and we can write the EPM in the general following form:

$$\text{(EPM)} \quad \begin{cases} (FC) \quad \underset{\textit{policy instrument}}{\textit{extremum}} \left\{ \sum_{\textit{time}} \text{discount factor} \left[ \text{objective function} \begin{pmatrix} \text{policy objectives} \\ \text{state variables} \\ \text{policy instruments} \\ \text{target weights} \\ \dots \end{pmatrix} \right] \right\} \\ (EM) \; [\text{state variables}] = F([\text{state variables}], [\text{policy instruments}]) \end{cases} \quad (1)$$

We can consider the model of economic policy as a problem of dynamic optimization, and the solution of this problem may take the form of a formula defining the optimal relation between the instruments and state variables and policy objectives. We have the following:

$$\text{(PR)} \qquad [\text{policy instruments}] = G([\text{policy objectives}], [\text{state variables}]) \qquad (2)$$

The important advantage of the discrete dynamic programming method is the possibility of its application to the determination of optimal control processes in which we take into account a random shock.

Below presented optimization problem of economic policy, which we will use to determine the expenditure, is the quadratic linear problem. In this problem the criterion function is a quadratic function and as a constraint we take into account linear dynamic model including the additive shock (Kendrick, 2005).

The Quadratic Linear Problem can be formulated following: (Benigno & Woodford, 2012; Ellison; Przybylska-Mazur, 2016): for each $t = 1,2,...,T$ we determine the control vector $U_t^*$ for which the function defined as:

$$G(X,U) = E_t\left(\sum_{t=0}^{T-1} \left(\left(X_t - X_t^o\right)^T V_t \left(X_t - X_t^o\right) + \left(U_t - U_t^0\right)^T Z_t \left(U_t - U_t^o\right)\right)\right) \qquad (3)$$

reaches a minimum with the constraint that is the linear dynamic model with additive shock. This model can be written in matrix form as follows (Kendrick and Amman, October 2011):

$$X_t = A \cdot X_{t-1} + B \cdot U_t + C \cdot \varepsilon_t \text{ for all } t = 1,2,...,T \qquad (4)$$

with the initial condition

$$X_0 = \tilde{X}_0 \qquad (5)$$

and with the restriction on an acceptable range of coordinates of control $U_{it} \in D_i$ for $t = 0,1,2,...,T-1$ and for each $i$,

where: $\varepsilon_t$ - random shock , $\varepsilon_t \sim IID(0, I)$, $T$ - planning horizon, $X_t$ - vector of state variables at time *t*, $U_t$ - control vector at time *t*, $X_t^o$ - vector of desired values of the state variables at time *t*, $U_t^o$ - vector of desired control values at time *t*, $\tilde{X}_0$ - given initial value of state vector, $A$ - matrix of state vector coefficients, $B$ - matrix of control vector coefficients, $B$ is multiplier matrix of impact of control variables, C - variance and covariance matrix of random shocks $\varepsilon_{t+1}$. $V_t$ are symmetric positive definite matrices of penalties of deviations of state variables from the desired values of state variables and $Z_t$ are symmetric positive definite matrices of penalties of deviations of control variables from the desired values. By $D$ we denote the following vector $D = \begin{bmatrix} D_1 & D_2 & \dots & D_p \end{bmatrix}^T$, $p$ is the number of instruments of fiscal policy.

About the selected methods of choice of the optimal monetary policy transmission horizon write Przybylska-Mazur (2013).

For determination of the optimal solution of economic policy model we define the Bellman function for the end section of the discrete trajectory of state in the following form (Bellman, 2010, Bellman, Dreyfus, 2015):

$$S(X_t) = \min_{\substack{U_k \in D \\ k = t, t+1, \dots, T-1}} \sum_{k=t}^{T-1} \left( \left(X_k - X_k^o\right)^T V_k \left(X_k - X_k^o\right) + \left(U_k - U_k^o\right)^T Z_k \left(U_k - U_k^o\right) \right) \quad (6)$$

Since the relationship is true:

$$S(X_t) = \min_{U_t \in D} \left( \left(X_t - X_t^o\right)^T V_t \left(X_t - X_t^o\right) + \left(U_t - U_t^o\right)^T Z_t \left(U_t - U_t^o\right) + S(X_{t+1}) \right) \quad (7)$$

then recursive Bellman equation for discrete process control is of the following form

$$S(X_t) = \min_{U_t \in D} \begin{pmatrix} \left(X_t - X_t^o\right)^T V_t \left(X_t - X_t^o\right) + \left(U_t - U_t^o\right)^T Z_t \left(U_t - U_t^o\right) + \\ + S(A \cdot X_{t-1} + B \cdot U_t + C \cdot \varepsilon_t) \end{pmatrix} \quad (8)$$

for $t = T-1, T-2, ..., 0$ with the final condition $S(X_T) = 0$.

This equation is the basis of discrete dynamic programming method, which reduces the determination of optimal controls sequence $\{U_t^*,\ t = 0,1,2,...,T-1\}$ to the determination of the individual controls $U_t^*$ from the equation recursive Bellman.

For the determination of optimal controls $U_t^*$ and the optimal values of the state vector $X_t^*$ we will use the algorithm of the stochastic dynamic programming method for the optimal control problem with discrete time (the algorithm backward), which consists the following steps described below (Bar-Shalom, 1982; Kendrick, 1982):

**1)** We substitute $t = T-1$ and we solve the optimalization problem for the final stage of the process

$$S(X_{T-1}) = \min_{U_{T-1}\in W}\left( E_{T-1}\left( \begin{matrix} \left(X_{T-1} - X_{T-1}^o\right)^T V_{T-1}\left(X_{T-1} - X_{T-1}^o\right)+ \\ +\left(U_{T-1} - U_{T-1}^o\right)^T Z_{T-1}\left(U_{T-1} - U_{T-1}^o\right)^T \end{matrix} \right)\right) =$$

$$= \min_{U_{T-1}\in W}\left( \begin{matrix} \int_{-\infty}^{+\infty} P(\varepsilon_{T-1})(\left(X_{T-1} - X_{T-1}^o\right)^T V_{T-1}\left(X_{T-1} - X_{T-1}^o\right)+ \\ +\left(U_{T-1} - U_{T-1}^o\right)^T Z_{T-1}\left(U_{T-1} - U_{T-1}^o\right)^T + \\ + S_{T-1}\left(A\cdot X_{T-2} + B\cdot U_{T-1} + C\cdot\varepsilon_{T-1}\right))d(\varepsilon_{T-1}) \end{matrix} \right) \qquad (9)$$

determining the optimal control for the last stage of the process $U_{T-1}^*$ as the function of the initial state of this stage $X_{T-1}$, thus $U_{T-1}^* = U^*(X_{T-1})$.

**2)** Using the function Bellman $S(X_{t+1})$ at $t-$th stage of the process, we solve the optimalization problem of this stage resulting from the recursive Bellman equation:

$$S(X_t) = \min_{U_t\in W}\left( E_t\left( \begin{matrix} \left(X_t - X_t^o\right)^T V_t\left(X_t - X_t^o\right)+ \\ +\left(U_t - U_t^o\right)^T Z_t\left(U_t - U_t^o\right)^T + S(f(X_t,U_t,\varepsilon_t)) \end{matrix} \right)\right) =$$

$$= \min_{U_t\in W}\left( \begin{matrix} \int_{-\infty}^{+\infty} P(\varepsilon_t)(\left(X_t - X_t^o\right)^T V_t\left(X_t - X_t^o\right)+ \\ +\left(U_t - U_t^o\right)^T Z_t\left(U_t - U_t^o\right)^T + S(f(X_t,U_t,\varepsilon_t))d(\varepsilon_t) \end{matrix} \right) \qquad (10)$$

determining the optimal control of $t-$th stage of the process $U_t^* = U^*(X_t)$ as the function of the initial state $X_t$ of this stage.

**3)** After reaching the initial stage for $t = 0$ we calculate the value of the optimal control $U_0^* = U^*(X_0)$ for this stage using the initial condition $X(0) = \breve{X}_0$.

**4)** Next we calculate the optimal sequence of controls on the basis of the relationships $U_t^* = U^*(X_t^*)$, where: $X_t^* = f(X_{t-1}^*, U_{t-1}^*, \varepsilon_{t-1})$, for $t = 1,2,...,T-1$.

Since the inflation rate, GDP growth and the unemployment rate are the main variables taken into account in the assumptions of the budget law we take into account these three variables in the article. Therefore $X_t = [\pi_t \quad Y_t \quad N_t]^T$, where $\pi_t$ - inflation rate, $Y_t$ - GDP growth and $N_t$ - unemployment rate. Considering in analyze only fiscal policy we take into account as the control variables: $W_t$ - budget expenditure and $P_t$ - budget revenue. Thus $U_t = [W_t \quad P_t]^T$.

Moreover, as vectors of desired values of the state variables and control variables we take $X_t^0 = \left[\pi_t^0 \quad Y_t^0 \quad N_t^0\right]^T$ and $U_t^0 = \left[W_t^0 \quad P_t^0\right]^T$, where $\pi_t^0$ - the inflation target, $Y_t^0$ - the potential GDP, $N_t^0$- the natural rate of unemployment, $W_t^o$ - the desired level of budget expenditure planned in the assumptions of the budget law generating a budget deficit no higher than 3% of GDP, $P_t^0$ - the desired level of budget revenues not less than assumed in the draft Budget Act. In addition, we assume the constant values of weight matrices $V_t = V = \begin{bmatrix} \lambda_\pi & 0 & 0 \\ 0 & \lambda_Y & 0 \\ 0 & 0 & \lambda_N \end{bmatrix}$, $Z_t = Z = \begin{bmatrix} \lambda_W & 0 \\ 0 & \lambda_P \end{bmatrix}$ for each $t$.

## 4 The empirical analysis

For the calculation of optimal values of control variables – the instruments of fiscal policy we use the annual inflation rate data (corresponding period of the previous year = 100), the GDP growth (annual data) and the annual unemployment rates (data published by Central Statistical Office, source: www.stat.gov.pl) and also the budget expenditures (annual data) and the budget revenues (annual data) (data published by the Ministry of Finance). For analysis we take into account the data for the Poland from the period 2004 to 2016. We take the planning horizon $T$ =3. As desired values of the state variables we take: the inflation target equals to 2.5%, the potential GDP and natural unemployment rate determined on the basis of the Hodrick – Prescott filter and the general government deficit equals to 3% of GDP from the convergence criteria. However the desired values of control vector are: the budget expenditures and the budget revenues planned in the draft Budget Act. As the forecasts of potential GDP growth, and the forecasts of unemployment rate for 2017 and 2018 we take the values published in the publication "Long-term Financial Plan for the years 2016-2019". The desirable budget revenue and desirable budget expenditure for the 2017 are taken from the draft Budget Act contained in the Budget Act for 2017. As desired budget revenue in 2018 is the value calculated on the basis of average value of improving tax collection contained in the publication "Long-term Financial Plan for the years 2016-2019." We assume in the analysis for each $t$ the following constant matrices $V_t = V = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{6} \end{bmatrix}$, because we put the greatest attention on the achievement of the highest growth and we take into account price stability that is the main objective of the strategy of direct inflation targeting. We study the expenditure rule, thus we assume the following weight matrices of control variables $Z_t = Z = \begin{bmatrix} \frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$ for each $t$.

The following table shows the optimal values of the control variables and optimal values of the state variables determined on the basis of the presented economic policy model.

The application of the optimal instrument of fiscal policy – the optimal budget expenditures and optimal budget revenues in the realization of fiscal policy allows to achieve the minimum deviation of state variables from the desired value of these variable, ie inflation from the inflation target, GDP growth from potential GDP growth and the unemployment rate from natural unemployment rate.

Based on this study we can conclude that the optimal values of control variables indicate to achieve the desired values of these variables. However, the practical application of obtained optimal instruments fiscal policy will lead to significant divergence between the optimal and desired values of state variables. This shows the necessary to change the desired values of the state variables in the years 2016-2018.

| **Horizon** *t* | **Year** | **Optimal values of** | |
|---|---|---|---|
| 0 | 2016 | control variable | |
| | | budget expenditure | 368548.53 |
| | | budget revenue | 313808.53 |
| 1 | 2017 | state variable | |
| | | inflation rate | 2.18 |
| | | GDP growth | 0.80 |
| | | uemployment rate | 4.47 |
| | | control variable | |
| | | budget expenditure | 384773.50 |
| | | budget revenue | 325428.00 |
| 2 | 2018 | state variable | |
| | | inflation rate | 2.72 |
| | | GDP growth | -0.96 |
| | | uemployment rate | 8.90 |
| | | control variable | |
| | | budget expenditure | 390773.50 |
| | | budget revenue | 328258.53 |
| 3 | 2019 | state variable | |
| | | inflation rate | 1.15 |
| | | GDP growth | -0.31 |
| | | uemployment rate | 12.40 |

**Table 1.** Optimal control values and optimal state variables.

## Conclusion

In the article we determined the fiscal policy rule. This rule is the solution of the Quadratic Linear Problem. We calculated the optimal values of fiscal instrument, that are the budget expenditures and the budget revenues. If the economy is regarded as a dynamic system with control, the application of the solution of Quadratic Linear Problem will help the economy develop in accordance with the desired path. The determined optimal fiscal policy rule has the positive impact on economy because it minimizes the deviation of inflation rate, GDP growth and unemployment rate from the desired values. In the simple proposed optimal fiscal rule, the budget expenditures and budget revenues depend on the inflation rate, the GDP growth and the unemployment rate. For the analysis ex ante we must use the forecasts of desired values of control variable and the forecasts of state variables, that can be determined different methods.

When we take into account the desired values of control vector and the desired values of state vector from the draft Budget Act and from the publication "Long-term Financial Plan for the years 2016-2019", we didn't received a satisfactory optimal values of the state variables. But for assumed desired values of the state and for desired values of control variables these optimal values allow to achieve the minimum deviation of state variables from the desired value of these variable, ie inflation from the inflation target, GDP growth from potential GDP growth and the unemployment rate from natural unemployment rate.

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