Risk measurement for goals realization in a household financial plan

Radosław Pietrzyk[1], Krzysztof Piontek[2], Paweł Rokita[3]

Abstract

This article presents a concept of household financial plan integrated risk measures with short-term and long-term approach to financial plan risk mitigation combined in one procedure. Short-term and long-term risk measures, based on household default probability, are introduced. Then, two approaches to their application in financial plan management process are proposed.

Keywords: *household, financial planning, risk measurement*

JEL Classification: D91, E21, J26, D10

1 Introduction

When constructing a financial plan for a household, it is very important to be able to incorporate risk, related to realization of its financial goals, into the model. The risk should be part of the plan choice criteria. The plan choice process should also be suited to natural psychological perspective of decision makers and it should be understandable for them.

People tend to think of their financial situation in rather a short-term perspective (Ballinger et al. 2003; Carbone and Hey, 2004; Carbone and Infante, 2012), whereas they need a life-long plan indeed. Amongst other reasons, the last is necessary for successful accomplishment of retirement goal. An approach that allowed to accommodate these perspectives would be thus welcomed. Moreover, it may be worth consideration to analyse risk aversion towards short-term and long-term threats separately. Treating risk aversion as one general characteristic of an investor, regardless the decision horizon, may be misleading.

Whereas there is a lot of discussion about intertemporal choice in respect of consumption in the existing literature, the solutions proposed there do not address the potential conflict between short-term risk minimization and minimization of the whole-plan risk, understood as the threat that the household will not be able to fully realize its life-long financial plan.

The work takes up the discussion about augmentation and modification of the known concepts in the area of household financial planning, to overcome the problem that has been

[1] Wroclaw University of Economics, Department of Financial Investments and Risk Management, Komandorska 118/120, 53-345 Wroclaw, Poland, e-mail: Radoslaw.Pietrzyk@ue.wroc.pl.
[2] Wroclaw University of Economics, Department of Financial Investments and Risk Management, Komandorska 118/120, 53-345 Wroclaw, Poland, e-mail: Krzysztof.Piontek@ue.wroc.pl.
[3] Wroclaw University of Economics, Department of Financial Investments and Risk Management, Komandorska 118/120, 53-345 Wroclaw, Poland, e-mail: Pawel.Rokita@ue.wroc.pl.

pointed out here. A proposal of risk measurement approach is presented. It combines risk of short- and long-term goals realization. Also such plan optimization constraint is proposed that household's preferences towards short- and long-term risk mitigation are incorporated.

2 Household financial planning model used

The discussion about household financial plan risk is based here on a conceptual framework for financial plan models, proposed by Jajuga et al. (2015). The discussion is, however, more general and not limited to the assumptions of this model only.

The framework is cash-flow based, discrete time, with a discrete space of scenarios. A scenario variable is a random vector with a discrete joint distribution. Elements of the random vector are risk factors. Each realization of the scenario variable is a scenario. A two-person household is assumed. The two persons are referred to as main household members. Though they are the only decision makers, cases with more persons (like children) are also covered by the model. Any number of financial goals is possible, with the retirement goal treated as a distinguished one – the one that must be set. For most households, this goal is unachievable without long-term planning (relatively big value and lack of post-financing possibility).

The model was developed in stages. The first, basic variant, called here the baseline model, assumes only two goals: retirement and bequest (if the household members show a bequest motive). Only two risk factors are taken into account there. These are dates of death of the main household members. The baseline model assumes only retirement investment.

The goal function of the optimization procedure is called value function of the household (Pietrzyk and Rokita, 2015b, 2016b). It is based on expected discounted utility of consumption and expected discounted utility of residual wealth. Its general idea builds on the classical concept by Yaari (1965), but it is suited to the discrete-time, discrete-space-of-states, two-person household model with a bequest motive. The household value function is maximized under two constraints: minimum consumption and the requirement that the pre-planned life annuity purchase expense is covered, fully and on time. The only decision variables in the financial plan optimization procedure are: consumption rate (of the household taken as a whole) in the first year of the plan and the proportion in which retirement investment contribution of the household is divided between systematic investment plans of each person.

The baseline model is constructed so that its augmentation is relatively easy. It may be extended to cover more financial goals, risk factors, as well as more ways of goal financing.

The general aim of the household is to realize all financial goals. The goal function of the optimization procedure is an augmented version of the one from the baseline model (Jajuga et al., 2015). Like in the baseline model, it is called value function of the household. Also new decision variables are introduced there. For example, the proportion in which each post-financeable goal is pre-financed (own contribution) and post-financed (debt) may be optimized. The set of boundary conditions consists now of: the minimum consumption constraint and the subset of constraints that all pre-planned cash outflows, that will result from accomplishment of the financial goals, must be covered fully and on time.

Originally, no bankruptcy level constraint or maximum indebtedness constraint was used in the Jajuga et al. (2015) model. If the result of optimization had shown an unacceptably high probability of bankruptcy, the decision makers would have had to review their goals, and then they would have started a new optimization procedure with a less ambitious goals set. Thus, risk was a criterion of plan choice in general, but not of the automated optimization procedure.

3 The role of integrated risk measures in financial plan management

One of key problems in the search for a life-long financial plan for a household is management of its risk. The easiest, but not recommended, approach is to analyse its different types separately. Risk steering methods that are suited to their specific risk types may be locally efficient, but life-long financial planning requires a more integrated approach.

The general scheme of risk management is more or less similar as for any other risky situation. That is, the main elements of *risk management process* are:

1) definition of risk management aims,
2) identification of risk types and risk factors,
3) risk measurement,
4) risk steering,
5) risk control.

The last (risk control) refers both to verification of risk steering effects and to overall control of all steps of the process.

In addition to the very risk management process, it is necessary to be aware of the role it plays in a more general process of financial plan management. The risk should be understood as the threat that the whole financial plan fails to be successfully accomplished. It may happen due to any of the possible reasons. This is an argument for integrated risk management.

In the proposal by Jajuga et al. (2015), integrated risk measurement is a part of household financial plan management process at the stage after plan optimization. Risk measures are used there to check if the plan that has already been optimized under a given set of constraints bears acceptable level of risk (where one of the optimization constraints is the requirement that the whole bunch of household financial goals should be realized). If the optimal plan is unacceptable, then the household members need to revise their goals and try to find a new optimal solution with the new set of boundary conditions. The reason for which the plan choice procedure is not fully automated is that no algorithm can autonomously change goals of the household. The concept of cyclical process of financial plan management and the position of risk measurement in this process is illustrated in **Fig. 1**.

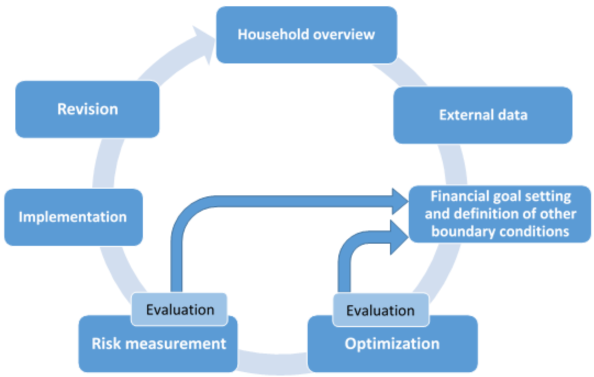

Fig. 1. Household financial plan management – a cyclical dynamic process.
Source: Jajuga, Feldman, Pietrzyk and Rokita (2015).

Nevertheless, some of the risk measures proposed there, as well as the new ones that are introduced in this article, allow to better integrate risk measurement with the plan management process. It would be close to an ideal situation if maximization of value function led automatically to an optimal trade-off between maximization of some measure of success and minimization of integrated risk measure. This would make the plan optimization problem analogous to portfolio optimization in the classical Markowitz world. At the present stage of model development, no such elegant proposal has been produced yet, nor the one that is proposed here fulfils this postulate. It is, however, possible to rebuild the optimization procedure so that a maximum acceptable level of risk is imposed there as a constraint.

Thanks to this solution, one may use the risk measure at an earlier step of financial plan management process, namely – on the stage of plan optimization. The risk management process may be then described like in the **Fig. 2**.

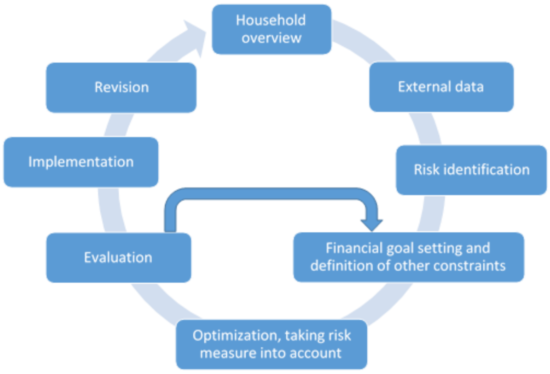

Fig. 2. Household financial plan management with risk incorporated into plan optimization procedure – a cyclical dynamic process.

Now, we need a measure of risk that may be conveniently used in the construction of a boundary condition. It must be also integrated and it should reflect threats to realization of the whole financial plan throughout the whole life of the household. By the term *integrated measure of risk* it is understood a measure that incorporates information on all types of risk and all risk factors that are taken into account in a given financial plan optimization model.

For the needs of this work, it is proposed to use measures based on household default probability (Jajuga et al., 2015; Pietrzyk and Rokita, 2016a). Section 3 describes this measure in more details, whereas section 4 discusses short-term and long-term measures based on it.

4 Household Default Probability (*HDP*)

Household default probability – a measure of risk proposed for the first time by Pietrzyk and Rokita (2015a) – is the probability that cumulated net cash flow falls below a critical shortfall level, called *default threshold*. In the model with a discrete number of scenarios and with discrete time, calculation of this probability consists in:

1) Analysing of each scenarios to check if cumulated net cash flow falls below the default threshold, at any moment of this scenario, since the start of the plan until the date when, under this scenario, the longer-surviving household member dies.
2) Marking, as a default scenario, each one in which cumulated net cash flow intersects the default threshold at least once.
3) Calculating probabilities of the default scenarios.
4) Summing probabilities of all default scenarios to obtain the probability that the household, having implemented this plan, defaults (*household default probability*).

A default threshold is determined at the plan starting moment (t_0) – for example, credit worthiness at t_0. Then, the default threshold changes with financial situation of the household.

Let us denote the multivariate random variable “scenario” with the symbol **Z** and some particular value of it – as $\mathbf{Z}^*$. Generally speaking, scenario is here a vector of risk factors that are taken into account in a given financial plan model. Generally speaking, scenario variable may be formally described as: $\mathbf{Z} = \left[X_1, X_2, ..., X_n\right]$, where the variables X_i are all random variables which are treated as risk factors under the assumptions of the model.

For example, in the baseline model, the scenario variable is bivariate and it consists only of dates of death. If we call one of the two main household members Person 1 and the second – Person 2, and denote the moment of death of Person 1 as D_1 and the moment of death of Person 2 as D_2, then the scenario variable is defined as: $\mathbf{Z} = \left[D_1, D_2\right]$. One particular scenario (a value or a realization of the random variable **Z**) may be then denoted as: $\mathbf{Z}^* = \left[D_1^*, D_2^*\right]$.

Definition of a default scenarios, that is – the ones under which cumulated shortfall exceeds the default threshold, may be formally described like in the formula (1):

$$\mathbf{T}^* = \mathbf{Z}^* : \underset{t=1,..,T_B^*}{\exists} CSp_t^{\left(\mathbf{Z}^*\right)} < DTh_t^{\left(\mathbf{Z}^*\right)}, \tag{1}$$

where: $CSp_t^{\left(\mathbf{Z}^*\right)}$ denotes cumulated net cash flow (cumulated surplus) at a moment t and under a scenario $\mathbf{Z}^*$, T_B^* is the moment of household end, under $\mathbf{Z}^*$, which is the moment of death of the household member who lives longer in this scenario: $T_B^* = \max\left\{D_1^*, D_2^*\right\}$, $DTh_t^{\left(\mathbf{Z}^*\right)}$ denotes the default threshold (its level as for the moment t, under $\mathbf{Z}^*$).

Probabilities of default are defined by the formula (2):

$$p_{\mathbf{T}_i^*} = \begin{cases} p_{\mathbf{Z}_i^*} & \text{if} \quad \underset{t=1,..,T_B^*}{\exists} CSp_t^{\left(\mathbf{Z}_i^*\right)} < DTh_t^{\left(\mathbf{Z}_i^*\right)} \\ 0 & \text{if} \quad \underset{t=1,..,T_B^*}{\forall} CSp_t^{\left(\mathbf{Z}_i^*\right)} \geq DTh_t^{\left(\mathbf{Z}_i^*\right)} \end{cases}, \tag{2}$$

where $p_{\mathbf{Z}_i^*}$ is probability of i-th scenario (i-th realization/value of the random variable **Z**).

Household default probability is a sum given in the formula (3)):

$$HDP = \sum_{i=1}^{n} p_{\mathbf{T}_i^*} . \tag{3}$$

The choice of *HDP* as the basic concept for the proposed risk measures is justified by the fact that it is relatively easy to impose constraints on it. A check of current value of the sums as defined in formula (3) may be performed in each iteration of the optimization procedure. It is possible to compare default probabilities for different plans. Thanks to this, a single default probability limit may be set for all suboptimal plans and the optimal one. For all of them, the level of default probability may be calculated and the constraint may be checked.

5 Short and long-term measures

The idea underlying short-term and long-term risk measures is that household members may have different preferences as to mitigation of risk in a short and long horizon. Having short-term and long-term measures defined, it is possible to add new constraints to the plan optimization procedure, which may allow household members to better express their preferences. Some acceptable levels of short-term and long-term risk may be set separately.

Let us now divide financial goals of the household into two groups:

- Group 1: *short term goals* –time until planned realization is shorter or equal to 1 year;
- Group 2: *long term goals* –longer time until planned realization (longer than 1 year).

Like for *HDP*, a default threshold is determined at the moment t_0 and it may change in time (e.g., different capability of financial shock absorbing in different life cycle phases).

For short term, scenarios in which cumulated shortfall intersects the default threshold in the first year are identified as short-term default scenarios – formally defined in the eq. (4):

$$\mathbf{T}^1 = \mathbf{Z}^* : CSp_1^{(\mathbf{z}^*)} < DTh_1^{(\mathbf{z}^*)}. \tag{4}$$

Probabilities of short-term default scenarios are defined by the formula (5):

$$p_{\mathbf{T}_i^1} = \begin{cases} p_{\mathbf{Z}_i^*} & \text{if} \quad CSp_1^{(\mathbf{z}_i^*)} < DTh_1^{(\mathbf{z}_i^*)} \\ 0 & \text{if} \quad CSp_1^{(\mathbf{z}_i^*)} \geq DTh_1^{(\mathbf{z}_i^*)} \end{cases}. \tag{5}$$

Then, probabilities of short-term default scenarios are summed (see the formula (6)). In this way, probability that any of the short-term default scenario realizes is obtained:

$$HDP_1 = \sum_{i=1}^{n} p_{\mathbf{T}_i^1} . \tag{6}$$

A long-term default scenario is a scenario in which no default is encountered in the first year, but there is incurred a default during whichever of the next periods – formula (7):

$$\mathbf{T}^2 = \mathbf{Z}^* : CSp_1^{(\mathbf{z}^*)} \geq DTh_1^{(\mathbf{z}^*)} \wedge \underset{t=2,..,T_B^*}{\exists} CSp_t^{(\mathbf{z}_i^*)} < DTh_t^{(\mathbf{z}_i^*)}. \tag{7}$$

Formula (8) gives a definition of a long-term default scenario probability:

$$p_{\mathbf{T}_i^2} = \begin{cases} p_{\mathbf{Z}_i^*} & \text{if} \quad CSp_1^{(\mathbf{z}_i^*)} \geq DTh_2^{(\mathbf{z}_i^*)} \wedge \underset{t=2,..,T_B^*}{\exists} CSp_t^{(\mathbf{z}_i^*)} < DTh_t^{(\mathbf{z}_i^*)} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

and the sum (9) is the long-term household default probability.

$$HDP_2 = \sum_{i=1}^{n} p_{\mathbf{T}_i^2} \; . \tag{9}$$

6 Application of short-term and long-term risk measures in plan optimization

Following a common-sense assumption that people would rather control risk of a short-term goal realization first, and then start to think about long-term ones, and being aware of the fact that, after all, a whole-life financial plan is necessary to be able to accomplish all financial goals, we propose some general workflow patterns. These are also ways in which the proposed risk measures may be implemented in the financial plan management process. They are based on the premise that household members may prefer to be able to express their short-term risk aversion separately from long-term risk aversion.

Let us consider two approaches. The first is very similar to the one by Jajuga et al. (2015). It does not involve any risk measures into financial plan optimization procedure. The only difference is that the household uses short-term and long-term measures to evaluate the result of optimization, besides the whole-plan risk measure.

Approach 1:

1. An optimal plan is found by an automated optimization formula that does not incorporate any risk measure.
2. After optimization, three risk measures are calculated for the optimal plan: *HDP*, *HDP*1 and *HDP*2.
3. On the basis of the three risk measures, the household makes decision if the plan may be accepted or some revision of financial goals and other boundary conditions is needed (**Fig. 1**). If the household chooses the revision variant, optimization is repeated after the revision. Since the revision means rethinking and resetting financial goals, a new plan is constructed indeed. The new optimization means, thus, searching for a new plan.

Approach 2

In this approach, some limits are imposed on default probabilities and these limits are used as constraints at the stage of plan optimization (**Fig. 2**). In this way, the measures become parts of the optimization procedure. Two variants of the approach 2 are discussed beneath.

Variant A (of the Approach 2)

In the first one, a weighted average of short-term and long-term default probability is calculated, where the weighting multiplier reflects household's preference of short-term risk mitigation versus long-term risk mitigation. The weighted sum is presented in the formula (10) and the parameter ξ denotes the preference to secure short-term goal realization. To that, the whole-plan default probability is calculated. Household members declare maximum acceptable values of the two probabilities – the weighted sum of the short-term and long-term default probabilities and the whole-plan household default probability (*HDP*). These two upper limits are used as constraints in the plan optimization procedure.

The new boundary conditions to be added are (formulas (10) and (11)):

$$\xi HDP_1 + (1-\xi) HDP_2 \le p^*, \tag{10}$$

$$HDP \le p^{**}. \tag{11}$$

Where upper limits of default probabilities (p^* and p^{**}) are declared by the household.

Variant B (of the Approach 2)

In the second variant, the household imposes two upper limits: one on the shorty-term household default probability and the second – on the long-term household default probability. The plan is optimized under the constraint that the two limits are held.

Beneath, short summary of the both variants is presented.

The new constraint to be added to the set of constraints is (formula (12)):

$$\left(HDP_1 \le p^{***}\right) \wedge \left(HDP_2 \le p^{****}\right), \tag{12}$$

where upper limits of default probabilities (p^{***} and p^{****}) are declared by the household.

It is important to point out that the Approach 1 gives different solutions than the Approach 2. In the first one, if the optimization result is rejected by the household due to high risk, financial goals are revised and optimization is performed with a new set of goals. In the second one, risk is measured within the optimization procedure and risk limits are treated as its constraints. If an optimal solution is feasible, it must be in compliance with pre-declared risk limits. Financial goal revision is, thus, unnecessary, unless no optimal solution exists.

7 Summary

All measures of risk discussed here are integrated, in the meaning that they incorporate information of all types of risk which are included in a given financial plan model. Moreover, measures proposed in section 4 allow to distinguish between short-term and long-term risk.

The proposed risk measures are based on household default probability (*HDP*), because it is relatively easy to impose a constraint on it.

The constraints on short-term and long-term risk are a way of reflecting household's aversion against risk in a short and long perspective. Moreover, setting of these constraints incorporates integrated risk measurement directly into plan optimization procedure. There have been no solutions proposed so far that would combine integrated risk measures with plan optimization, taking, moreover, short-term and long-term risk aversion into account.

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