Overlapping generation models with housing: impact of the key parameters on the models' outcomes

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Abstract

Overlapping generation models (OLG) with housing are used to analyze interactions between households of different age, housing market and the main macroeconomic aggregates like production, consumption or interest rate. In the paper, we study sensitivity of these categories to changes in the main parameters of the models, especially the discount factor, the risk aversion parameter, demographic characteristics and housing market parameters. The baseline model is calibrated to match the key features of the Polish economy.

Keywords: *overlapping generations, housing, calibration, wealth, debt*

JEL classification: C63, E20

1 Introduction

Overlapping generation models developed by Diamond (1965) and Auerbach and Kotlikoff (1987) become increasingly popular recently as a tool to study interactions between macroeconomic aggregates and households in different phases of life-cycles. They are used to study the effects of fiscal and monetary policy (Auerbach and Kotlikoff, 1987; Kindermann and Krueger, 2014; Doepke et al., 2015) as well as social security on income and wealth inequality, consumption and material deprivation of households (Gertler, 1999; Hairault and Langot, 2007; Cheron et al., 2011; Acedański, 2016; Bielecki et al., 2015a, 2015b). Recently, the models are augmented with housing market (Chen, 2010; Rubaszek, 2012) because of its important role in business cycles, as shown by the last financial crisis, and the fact that majority of households' wealth is stored in real estates. Housing also plays an indispensable role in modelling households' debt as mortgage loans dominate households' liabilities in terms of their value.

Unfortunately, the popularity of the discussed models is limited to some extent by their computational complexity. Deriving household's consumption and housing decision rules under rational expectations requires solving fixed points problems with nested stochastic, dynamic programs with finite horizon. Such problems can be solved only approximately with computationally-intensive numerical procedures. The time-consuming computations result in difficulties with the correct calibration of the models' parameters as the approximations have

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to be executed many times for different sets of parameter values to match the targeted empirical characteristics with the analyzed model.

This paper is intended to provide some form of guidance on the role of the typical OLG model's parameters for shaping the most important characteristics like interest rate, housing characteristics and age profiles of selected variables which should facilitate the calibration process. We investigate sensitivity of the mentioned characteristics to the changes in the main parameters of the model. The baseline model's calibration aims at matching the key characteristics of Polish economy.

The paper is organized as follows. Section 2 contains the description of the model. The baseline calibration is discussed in section 3. The results of our sensitivity checks are presented in section 4.

2 Model

We study the open economy overlapping generations model with housing closely related to Chen (2010) and Rubaszek (2012). There are four types of agents in the model: households, firms, financial intermediaries and government which are described in more details below.

We use the following notation: capital letters refer to the aggregate variables, an individual household's characteristics are described by small letters and primes denote the next-period variables.

The economy is inhabited by a continuum of households that differ in terms of age *j*, stochastic idiosyncratic labor productivity $e \in \{e_1, e_2, \ldots, e_I\}$ and asset holdings. Households store their current wealth in form of deposits a or housing h_o. They enter labor market at age 25 with no wealth, work until 64 then retire and live up to 84 at most. Because we focus on the low-frequency movements in the interest rates only twelve cohorts are considered: $j \in \{25, 30, \ldots, 80\}$. The life span is stochastic and the probability that a household at age j survives to age $j+5$ is denoted by s_j. The share of the cohorts j to $j+4$ in the total population is μ_j whereas the share of a labor productivity group $i \in \{1, 2, ..., I\}$ is denoted by μ_{ei}.

Households derive utility from consumption of nondurable goods *c* and housing services *h*. The households either own or rent houses and the flows of housing services in both cases are denoted by h_o and h_r, respectively. The momentarily utility function of a household is given by:

$$u(c,h) = \frac{(c^\theta h^{1-\theta})^{1-\eta} - 1}{1-\eta} \tag{1}$$

where $h = h_o$ if a household owns a house, $h = h_r$ if it rents one or $h = \underline{h}$ otherwise, where $\underline{h}$ denotes some small positive level of housing services, η is the relative risk aversion parameter and θ represents the share of nondurable goods consumption in the utility function. Households also care about next generations and derive utility from leaving bequests b in the following form:

$$u_b(b) = \kappa \frac{b^{1-\eta} - 1}{1-\eta}, \tag{2}$$

where κ governs the strength of the bequest motive.

Young households work and earn income that depends on their age and idiosyncratic labor productivity. Their net earnings are $(1-\tau)e_i\varepsilon_j W$, where τ represents the tax rate, W denotes the average wage in the economy and ε_j is deterministic, age-related productivity component. Retired households receive pensions that are constant across households and independent of their earnings' history[2]. The pension is equal to the fraction χ of the average wage in the economy. Labor-related income of a household can be written as:

$$inc = \begin{cases} (1-\tau)e_i\varepsilon_j W & \text{if} \quad j < 65 \\ \chi W & \text{if} \quad j \geq 65 \end{cases}. \tag{3}$$

A representative firm produces the final nondurable good Y using the Cobb-Douglas technology:

$$Y = K^{\alpha} L^{1-\alpha}, \tag{4}$$

where K is the aggregate physical capital and $L = \sum_{j<65} \varepsilon_j \mu_j \sum_i e_i \mu_{ei}$ is the aggregate effective labor input. The productive capital is rented from the financial intermediaries whereas labor is provided by households. Because the production sector is perfectly competitive the capital demand of the firm is set so that the marginal product of capital matches the gross interest rate $R + \delta$ and the aggregate wage is equal to the marginal product of labor:

$$W = (1-\alpha) K^{\alpha} L^{-\alpha}, \tag{5}$$

$$R + \delta = \alpha K^{\alpha-1} L^{1-\alpha} \tag{6}$$

where δ denotes the depreciation rate of physical capital. The final good can be consumed or invested in either physical capital or housing.

[2] This assumption is common in the literature and is made for computational tractability of the model.

The perfectly competitive financial sector collects deposits from households and foreign investors. The aggregate net value of foreign deposits A^* is equal to:

$$A^* = \xi(R - R^*)Y, \tag{7}$$

where R^* is the world interest rate and the parameter ξ measures the degree of the economy's openness (autarky if $\xi = 0$ and perfect international financial markets if $\xi = \infty$). This parameter determines also the sensitivity of the domestic interest rate to the world's one.

The financial sector grants mortgage loans to homeowners and buys physical capital and rental housing. Capital is then rented to the final good producers and to the households that are unable to buy their own house. It is assumed that the interest rates on deposits, mortgage loans and physical capital are equal. Solving the profit maximization problem of a representative financial intermediary and assuming that the profit in the equilibrium is zero give the rental price on housing R_r:

$$R_r = \frac{R' + \delta_r}{1 + R'} \tag{8}$$

where δ_r represents the depreciation of rental housing.

The government's role is twofold. First, it levies taxes on labor to finance pensions. The government's budget is balanced so the tax rate is equal to:

$$\tau = \frac{\chi}{L} \sum_{j \geq 65} \mu_j . \tag{9}$$

Secondly, following the literature it is assumed that the government collects bequests and distributes them equally among the living households. These lump-sum transfers are denoted by tr.

A household maximizes its expected discounted lifetime utility. It takes its current deposit stock a, housing stock h_o, idiosyncratic productivity e and age j as givens and makes a housing tenure decision first. Then, a homeowner decides on the house size, nondurable consumption level and the financial assets level (deposit or debt). A renter chooses housing services offered by the financial intermediaries, the nondurable consumption level and the deposit level. It is not allowed to borrow.

Value functions of a household that chooses to own or rent a house are denoted by V_o and V_r, respectively. The decision problem of a household that decides to own a house can be written recursively:

$$V_o(a, h_o, e, j) = \max_{c, h'_o, a'} \left\{ u(c, h_o) + \beta \left[s_j E(V(a', h'_o, e', j+1) \mid a, h_o, e, j) + (1 - s_j) u_b(b') \right] \right\} \tag{10}$$

subject to:

$$(1+R)a+(1-\delta_o)h_o+inc+tr=a'+h'_o+\phi(h_o,h'_o)+c, \tag{11}$$

$$a'\geq-(1-\gamma)h'_o, \tag{12}$$

$$b'=(1+R)a'+(1-\delta_o)h'_o. \tag{13}$$

where $V=\max\{V_o,V_r\}$ with V_r defined below, δ_o denotes the depreciation rate of owned housing and ϕ represents transaction costs associated with a change in the size of the owned house:

$$\phi(h,h')=\begin{cases}0 & \text{if} \quad h=h',\\ \phi(h+h') & \text{if} \quad h\neq h'.\end{cases} \tag{14}$$

Equation (11) is the budget constraint of a household. Equation (12) represents the downpayment borrowing constraint which states that a household can borrow up to $(1-\gamma)\cdot 100\%$ of the owned house value. Finally, equation (13) defines the value of a bequest.

The value function of a household which chooses to rent a house takes the following form:

$$V_r(a,h_o,e,j)=\max_{c,h'_r,a'}\left\{u(c,h'_r)+\beta\left[s_j E(V(a',0,e',j+1)\mid a,h_o,e,j)+(1-s_j)u_b(b')\right]\right\} \tag{15}$$

subject to:

$$(1+R)a+(1-\delta_o)h_o+inc+tr=a'+R_r h_r+\phi(h_o,0)+c, \tag{16}$$

$$a'\geq 0. \tag{17}$$

3 Calibration

The parameters are calibrated to match the main characteristics of Polish economy. The real risk free rate in the US is used as a proxy for the world interest rate. As already mentioned, one period in the model corresponds to five years. The values of the parameters are presented in table 1.

The steady state level of the world interest rate is set to $\overline{R}^*=3.65\%$ per annum which is equal to the mean real risk free rate in the US in period 1947–2016 according to the data provided by R. Shiller's website. The parameter ξ associated with the degree of the economy openness is set to 1.3 to match the average net foreign asset position A^*/Y of Poland which is roughly -55%.

Symbol	Description	Value
$\overline{R}^*$	Steady state level of the world interest rate (annualized)	3.65%
ξ	Openness of the economy	1.3
s_j	Survival probabilities	CSO (2016)
e , P_e	Idiosyncratic productivity shocks and transition matrix	Rubaszek (2012)
β	Discount coefficient (annualized)	0.971
η	Risk aversion	3
θ	Share of nondurable consumption in the utility function	0.75
κ	Strength of the bequest motive	15
χ	Pension replacement rate	0.6
α	Capital share in the production function	0.3
δ	Capital depreciation rate (annualized)	0.08
δ_o	Owned housing depreciation rate (annualized)	0.013
δ_r	Rented housing depreciation rate (annualized)	0.025
γ	Downpayment ratio	0.15
ϕ	Transaction costs	0.075
h_{min}	Minimum house size	0.5
h_{max}	Maximum house size	2.25
$\underline{h}$	Minimum housing consumption	$0.1 \cdot h_{min}$

Table 1. Baseline calibration of the model.

Twelve cohorts are considered. Survival probabilities are based on Polish unisex life tables published by Central Statistical Office (2016). The annual discount coefficient is equal to 0.971 to match the average interest rate spread between Poland and the US of 2.8% per annum. A fairly standard risk aversion parameter η = 3 is used. The share of nondurable consumption in the utility function is set to 0.75 following Rubaszek (2012). The parameter κ = 15 that governs strength of the leaving bequest motive is determined to match the age profiles of the fraction of households with debt and the average debt value in Poland according to data provided by the National Bank of Poland (2015), particularly in the oldest cohorts. The pension replacement rate parameter $\chi = 0.6$ matches the ratio observed in Poland in recent years. Finally, the idiosyncratic productivity levels together with the transition matrix for the shock are taken from Rubaszek (2012).

We use a standard values of the technology parameters for Poland setting the capital share $\alpha = 0.3$ and the annual capital depreciation rate $\delta = 0.08$. The depreciation rates for the housing market follow Chen (2010): $\delta_o = 0.013$ and $\delta_r = 0.025$. As a result of the difference in the depreciation rates owned housing is cheaper than rented which is another incentive for households to buy rather than rent a house.

The house sizes h that are available to buy or rent are discretized. The set of sizes takes the form $h \in \{0, h_{min}, ..., h_{max}\}$, where the sizes between h_{min} and h_{max} are equally spaced. These values together with the mortgage downpayment ratio γ, the transaction costs parameter ϕ and $\underline{h}$ are jointly determined to match the age profiles of the fraction of households with debt and the average debt value.

Table 2 contains the key model's characteristics for the baseline calibration and their empirical counterparts for Poland. The real interest rate in the model coincides with the mean real interest rate based on data provided by OECD. Average total wealth relative to mean annual income of households in the model exceeds the value observed in the data. However, the specification of the model does not allow to simultaneously match the interest rate and the wealth level as they are tightly linked. Similar result is observed for housing wealth. The model overestimates the homeownership rate but the data collected by NBP (2016) does not account for social housing as pointed out by Rubaszek (2012). The fraction of households in debt and the average value of debt, where the latter is expressed relative to households' annual income, in the model are slightly lower compared to the data. Finally, the model generates unrealistically low wealth inequality. This results from the assumptions that households enter the labor market with no wealth and receive equal pensions. Moreover, the low number of cohorts limits the possibility to generate substantial wealth inequality.

Characteristics	**Source**	**Data**	**Model**
Real interest rate [%]	OECD (2016)	6.4	6.4
Average total wealth	NBP (2015), CSO (2016a)	6.8	8.6
Homeownership rate [%]	NBP (2015)	77.4	90.9
Average housing wealth	NBP (2015), CSO (2016a)	5.4	6.5
Fraction of households in debt [%]	NBP (2015)	37.0	33.9
Average debt	NBP (2015), CSO (2016a)	0.84	0.67
Gini coefficient for wealth	NBP (2015)	0.579	0.368

Table 2. The model's fit for the baseline calibration.

4 Results

In table 3, we report changes in the mean values of the selected characteristics caused by changes in the parameters of the model. We separately study the effects for eight different parameters. For each parameter, we always consider two reasonable alternative values: the lower and the higher than the baseline one. The rental housing depreciation rate δ_r, transaction costs ϕ and the economy openness parameter ξ have limited impact on the considered variables. The effects generated by the other parameters are discussed in more details below.

Calibration	Interest rate	Wealth	Homeown. rate	Housing wealth	Househol. in debt	Average debt	Gini for wealth
Baseline	**6.4**	**8.6**	**90.9**	**6.5**	**33.9**	**0.67**	**0.368**
$\alpha = 0.25$	-0.7	-4.7	5.5	-1.5	5.1	4.5	-0.3
$\alpha = 0.4$	1.6	8.1	-19.2	1.5	-11.6	35.8	8.2
$\beta = 0.961$	0.8	-7	-6.4	-3.1	-1.1	23.9	2.4
$\beta = 0.981$	-0.7	5.8	3.1	3.1	0.8	-9	-0.5
$\delta_r = 0.02$	-0.1	0	0.3	0	0.2	-4.5	0
$\delta_r = 0.03$	-0.1	0	0.5	0	0.4	-4.5	-0.3
$h_{min} = 0.25$	-0.6	-4.7	9.1	-10.8	-4.2	-49.3	5.2
$h_{min} = 0.75$	0.1	1.2	-23.9	3.1	-3.1	11.9	8.4
$\gamma = 0.1$	0.3	-1.2	-5.1	0	-2	43.3	6
$\gamma = 0.2$	-0.1	0	0.5	-1.5	0	-7.5	-0.5
$\phi = 0.1$	-0.1	0	0.8	-1.5	0.6	-3	-0.8
$\phi = 0.05$	0	0	-0.1	1.5	0	1.5	0.8
$\theta = 0.65$	1.4	12.8	5.5	26.2	4.7	50.7	0.5
$\theta = 0.85$	-1.6	-14	-17.7	-30.8	-13.6	-62.7	7.6
$\xi = 1$	-0.2	-1.2	1.6	0	1.9	4.5	-0.3
$\xi = 2$	0.3	1.2	-0.8	0	-1.3	-6	0

For the percentage variables (interest rate, homeownership rate, households in debt) simple differences in percentage points from the baseline value ($\Delta X = X - X_b$, where X_b denotes a value for the baseline calibration) are reported; for the other characteristics relative percentage differences are calculated ($\Delta X_{rel} = 100 \cdot (X / X_b - 1)$)).

Table 3. Results of the simulations under different parametrizations.

The capital share α plays the important role for almost all studied characteristics. It is positively related to the interest rate, wealth and wealth inequality. The rise in capital share significantly decreases the homeownership rate and the fraction of households in debt. Also the average value of debt is significantly nonlinearly affected by the changes in α.

The role of the discount factor β for macroeconomic characteristics is well known in the literature. It is inversely related to the interest rate and wealth inequality. Additionally, we show that the lower discount factor significantly increases average debt and reduces the homeownership rate. The latter characteristic is also sensitive to the changes in the minimum house value h_{min} which also affects housing wealth, the average debt level and, in a nonlinear fashion, the Gini coefficient for wealth. The downpayment constraint γ plays the important role for the mean value of debt. The looser constraint the higher the debt is which also leads to higher wealth inequality.

Finally, almost all characteristics are sensitive to changes in the share of nondurable consumption in the utility function θ. The rise in θ reduces wealth, homeownership rate, housing wealth, fraction of households in debt as well as the mean level of debt. It also increases wealth inequality considerably.

Conclusion

In the paper, we considered the standard OLG model with housing for Polish economy. We studied sensitivity of the selected characteristics to small changes in the key parameters to identify those who have the largest impact on the outcomes. We found that the share of nondurable consumption in the utility function plays the important role in shaping both the macro and the housing characteristics. Furthermore, the capital share and the minimum house size turned out to be important as well.

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