# Application of selected dynamic model to the analysis of the impact balanced budget rule on the economy

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**Abstract**

The fiscal rules are helpful to conduct the sustainable fiscal policy, that is a necessary condition for achieving the sustainable economic development in the long term. Since the government should seek to maintain a balanced budget, the balanced budget rule may be helpful in the conduct of fiscal policy. In addition, one of the Maastricht criteria is the reduction of the general government deficit to 3% of GDP. Therefore, the balanced budget rule include often the limit of an acceptable level of budget deficit in relation to GDP. Using the selected dynamic model for fiscal policy and monetary policy and the control theory in article we determine the optimal values of fiscal policy instruments and of monetary policy instrument. The objective of the article is the presentation of the optimal rules in context of the benefits and the costs for the economy resulting from fiscal decision-making based on balanced budget rule.

***Keywords:*** *balanced budget rule, fiscal policy model, control theory, benefits and costs for the economy*
***JEL Classification:*** E62, C54, C61

## 1. Introduction

The fundamental importance of the implementation of economic policy in the short and medium term, the so-called macroeconomic policies have the monetary policy and the fiscal policy. The requirement to reduce public debt to 60% of GDP and the budget deficit to 3% of GDP introduced the Maastricht Treaty that was signed by EU countries in 1992. The Stability and Growth Pact has guarantee that Member States of the European Union will keep public finances in equilibrium and these countries will coordinate the fiscal policy. Therefore, we must control the annual public debt and general government deficit. It is worth also have regard to the annual changes in fiscal policy that could have a beneficial effect on the performance of stabilization policy, which aims to mitigate the fluctuations in economic activity caused by the change of phases of the business cycle. Therefore, we should have into account the optimal monetary and fiscal policy transmission horizon. We can use different methods to determine this horizon (see Przybylska-Mazur, 2013).

The objective of the article is the determination of fiscal and monetary rule as the solution of Quadratic Linear Tracking Problem with the constraints of the dynamic model for fiscal

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and monetary policy. The objective of this article is also the determination of these optimal rules in terms of benefits and costs for the economy resulting from the making decision on the basis of the balanced budget rule.

The first part of this article discusses general fiscal policy and monetary policy based on rules and we present the benefits and the costs for the economy resulting from the making decision on the basis of optimal balanced budget rule. The second part of this article presents a dynamic model describing the dynamics of tax rates and interest rate. We present also the optimal control problem and we determine the optimal fiscal and monetary policy rules that are the solution of the quadratic linear problem. These rules are the feedback rules. The application of these rules allows the economy to develop according to the desired path. In the last part of the article we present the results of empirical analysis for Poland. Article summarizes the conclusion.

## 2. Policy based on rules and its impact on the economy

The fiscal policy involves government decisions on the budget deficit and/or the public debt and/or the size and structure of public expenditure. One of the tools of fiscal policy are tax rates. However the basic tool of the monetary policy is interest rate.

One of the ways of decision making is the decisions based on rules. When we conduct the fiscal policy based on rules it is strengthened the fiscal policy caution and objectivity in the implementation of this policy. Fiscal rules including the balanced budget rule are an effective tool for limiting the ability to generate excessive deficits. Fiscal rules have a significant impact on the economy. Therefore, there are many benefits for the economy resulting from the use of fiscal rules in particular the balanced budget rule. One of the benefits is the creation of favorable conditions to raising potential GDP growth. However, in order to the rules fulfill their corrective function is not enough to introduce them. Effective fiscal rules should cover the entire public finance sector and they should be relatively simple and flexible. According to the International Monetary Fund the effective balanced budget rule help in economic stability and reduction of debt. Thus, the benefits resulting from the use of balanced budget rule are economic stabilization and reduction of debt. Fiscal sustainability, as an integral part of macroeconomic stability will strengthen the protection of the economy against various types of shocks. If it is necessary a fiscal impulse to stimulate economic conditions, it should be possible emergence of budget deficit. The fiscal impulse can not lead to a sustainable public debt. Therefore, the using fiscal policy model should lead to long-term stable economic growth. In countries belonging to the monetary union, where a tool for stabilizing the

economy is fiscal policy, balance of the structural balance is very important for the effective functioning of these countries. The criteria of the Maastricht Treaty concerning also on the public finances say that the public debt can not exceed 60 percent of GDP, however the budget deficit can not exceed 3 percent of GDP.

The costs of implementation of the balanced budget rule will occur primarily in the short term, because it may be necessary increase of taxes and the reduction of the budget expenditures. This will have primarily an impact on the citizen's welfare, but it is also not conducive to stimulation of the economy.

Therefore, the important issue to the solution is the determination of the optimal fiscal and monetary policy rules, because in order to achieve the goals is essential coordination of fiscal policy and monetary policy. The application of these rules allows the economy to develop according to the desired path. In order to determine these rules we applied the control theory and a selected form of dynamic model of fiscal and monetary policy

## 3. Dynamic model and the Quadratic Linear Problem

Many of the problems in the economy can be modeled with the use the dynamic models. In the article to determine the fiscal and monetary policy feedback rules we take into account the dynamic model that can be written in matrix form as follows (Kendrick and Amman, 2011; Svensson, 1998):

$$X_{t+1} = A \cdot X_t + B \cdot U_t \text{ for all } t = 0,1,..., N-1, \tag{1}$$

with the initial condition:

$$X_0 = \tilde{X}_0 \tag{2}$$

where $X_t$ – vector of state variables at time *t*, $U_t$ – control vector at time *t*, $X_t^*$ – vector of desired values of the state variables at time *t*, $U_t^*$ – vector of desired control values at time *t*, $\tilde{X}_0$ – given initial value of state vector, the state vector at time $t = 0$, $A$ – matrix of state vector coefficients at time *t*, $B$ – matrix of control vector coefficients at time *t*, that is multiplier matrix of impact of control variables, $V_t$ is symmetric positive definite matrix of penalties of deviations of state variables from the desired values of state variables and $S_t$ is symmetric positive definite matrix of penalties of deviations of control variables from the desired values.

When we makes economic and investment decision we may take into account the inflation (see Hadaś-Dyduch, 2014). Thus, in this article we take into consideration the inflation rate

$\pi_t$ as one of the state variable. As other state variables we consider: the GDP growth $Y_t$ and the ratio of general government deficit to GDP $D_t$, therefore $X_t = [\pi_t \quad Y_t \quad D_t]^T$, while the control variables are the interest rate $i_t$, the tax rates: the PIT rate (basic rate) $\tau_t$ and the VAT rate (basic rate) $v_t$, thus $U_t = [i_t \quad \tau_t \quad v_t]^T$. Moreover, as vectors of desired values of the state variables and control variables we take $X_t^* = [\pi_t^* \quad Y_t^* \quad D_t^*]^T$ and $U_t^* = [i_t^* \quad \tau_t^* \quad v_t^*]^T$, where $\pi_t^*$ – the inflation target, $Y_t^*$ – the potential output, the desirable general government deficit equals to 3% (of GDP), $i_t^*$ – the natural interest rate, $\tau_t^*$, $v_t^*$ – the desirable tax rates calculate from linear trends. Furthermore, we assume $V_t = \begin{bmatrix} \lambda_{\pi t} & 0 & 0 \\ 0 & \lambda_{Y t} & 0 \\ 0 & 0 & \lambda_{D t} \end{bmatrix}$, $S_t = \begin{bmatrix} \lambda_{it} & 0 & 0 \\ 0 & \lambda_{\tau t} & 0 \\ 0 & 0 & \lambda_{v t} \end{bmatrix}$.

Now we present the quadratic linear problem, that we use to determine the fiscal rule. In the quadratic linear problem criterion function (Benigno and Woodford, 2012) is the quadratic function, but as a limiting conditions we take the linear equation system. If we take into account in the analysis that the values of analyzed economic variables are carried out in accordance with the desired trajectory, we should consider the so-called tracking problem. Thus, in article we determine the fiscal policy rules as the solution of Quadratic Linear Tracking Problem, that can be formulated following: for each $t = 0,1,..., N-1$ we determine the control vector $U_t$ for which the function being the cost-to-go (Kendrick, 1982) defined as:

$$J = \frac{1}{2}\left(X_N - X_N^*\right)^T \cdot V_N \cdot \left(X_N - X_N^*\right) + \\ + \frac{1}{2}\sum_{t=0}^{N-1}\left(\left(X_t - X_t^*\right)^T \cdot V_t \cdot \left(X_t - X_t^*\right) + \left(U_t - U_t^*\right)^T \cdot S_t \cdot \left(U_t - U_t^*\right)\right), \tag{3}$$

reaches a minimum.

The optimal linear feedback rule we obtain as the solution of the problem (1)-(3) (Ellison, 2004). This rule is given by the following formula (Kendrick and Amman, 2011):

$$U_t = G_t \cdot X_t + g_t \text{ for all } t = 0,1,..., N-1 \tag{4}$$

where $G_t$ – the feedback gain matrix at time *t*, $g_t$ – the feedback parameter vector at time *t*, which are calculated vector from the following formulas:

$$G_t = -\left(B^T \cdot K_{t+1} \cdot B + S_t^T\right)^{-1} \cdot B^T \cdot K_{t+1} \cdot A, \tag{5}$$

$$g_t = -\left(B^T \cdot K_{t+1} \cdot B + S_t^T\right)^{-1} \cdot \left[B^T \cdot p_{t+1} - S_t \cdot U_t^*\right], \tag{6}$$

matrix $K_t$ and vector $p_t$ fulfill the following Riccati equation (Ferrante and Ntogramatzidis, 2013) for each $t = 1,2,..., N-1$:

$$K_t = V_t + A^T \cdot K_{t+1} \cdot A - A^T \cdot K_{t+1} \cdot B \cdot \left(B^T \cdot K_{t+1} \cdot B + S_t^T\right)^{-1} \cdot B^T \cdot K_{t+1} \cdot A \tag{7}$$

$$p_t = A^T \cdot p_{t+1} - V_t \cdot X_t^* - A^T \cdot K_{t+1} \cdot B \cdot \left(B^T \cdot K_{t+1} \cdot B + S_t^T\right)^{-1} \cdot \left(B^T \cdot p_{t+1} - S_t \cdot U_t^*\right) \tag{8}$$

whereas for $t = N$:

$$K_N = V_N, \tag{9}$$

$$p_N = -V_N \cdot X_N^*. \tag{10}$$

Therefore, taking into account the matrix representation of feedback rule (4), we can write the fiscal rules in the following form:

$$\tau_t = G_{21,t} \cdot \pi_t + G_{22,t} \cdot Y_t + G_{23,t} \cdot D_t + g_2, \tag{11}$$

$$v_t = G_{31,t} \cdot \pi_t + G_{32,t} \cdot Y_t + G_{33,t} \cdot D_t + g_3, \tag{12}$$

and the monetary rule in the following form:

$$i_t = G_{11,t} \cdot \pi_t + G_{12,t} \cdot Y_t + G_{13,t} \cdot D_t + g_1. \tag{13}$$

The fiscal policy and monetary policy feedback rules show the dependence policy instrument on the inflation rate, the GDP growth and general government deficit.

## 4. The empirical analysis

For the calculation of optimal fiscal and monetary policy rules we use the annual inflation rate data (corresponding period of the previous year = 100), the GDP growth (annual data) and the general government deficit (annual data) (data published by Central Statistical Office, source: www.stat.gov.pl), the annual average of reference rate (data published by the NBP, source: www.nbp.pl) and the basic values of PIT rate and VAT rate (annual data) (data published by the Ministry of Finance). For analysis we take into account the data for the Poland from the period 2004 to 2015 year. As desired values of the state variables we take: the inflation target, the potential GDP determined on the basis of the Hodrick–Prescott filter and the general government deficit equals to 3% of GDP from the convergence criteria. However the desired

values of control vector are: the natural interest rate determined on basis of Hodrick–Prescott filter and desired values of tax rates: PIT rate and VAT rate calculated from linear trend function. Furthermore, we assume the constant weight values for each $t$, thus $V_t = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$ and we assume $S_t = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$ for each $t$.

On the basis of these data we calculate the optimal values of PIT rate, VAT rate and interest rate. In the Table 1 we summarize the optimal tax rates with the real values.

| **Year** | **The optimal values of PIT rate** | **The real values of PIT rate** | **The optimal values of VAT rate** | **The real values of VAT rate** |
|---|---|---|---|---|
| 2003 | 18.51 | 19.00 | 22.66 | 22.00 |
| 2004 | 19.01 | 19.00 | 23.21 | 22.00 |
| 2005 | 18.92 | 19.00 | 22.29 | 22.00 |
| 2006 | 18.99 | 19.00 | 22.15 | 22.00 |
| 2007 | 18.98 | 19.00 | 22.17 | 22.00 |
| 2008 | 18.58 | 19.00 | 22.29 | 22.00 |
| 2009 | 18.32 | 18.00 | 23.15 | 22.00 |
| 2010 | 18.27 | 18.00 | 23.02 | 22.00 |
| 2011 | 18.44 | 18.00 | 22.97 | 23.00 |
| 2012 | 18.12 | 18.00 | 22.27 | 23.00 |
| 2013 | 17.74 | 18.00 | 22.03 | 23.00 |
| 2014 | 17.72 | 18.00 | 22.41 | 23.00 |

**Table 1.** The optimal and the real values of PIT rate and VAT rate.

The obtained results we present also in the figures below.

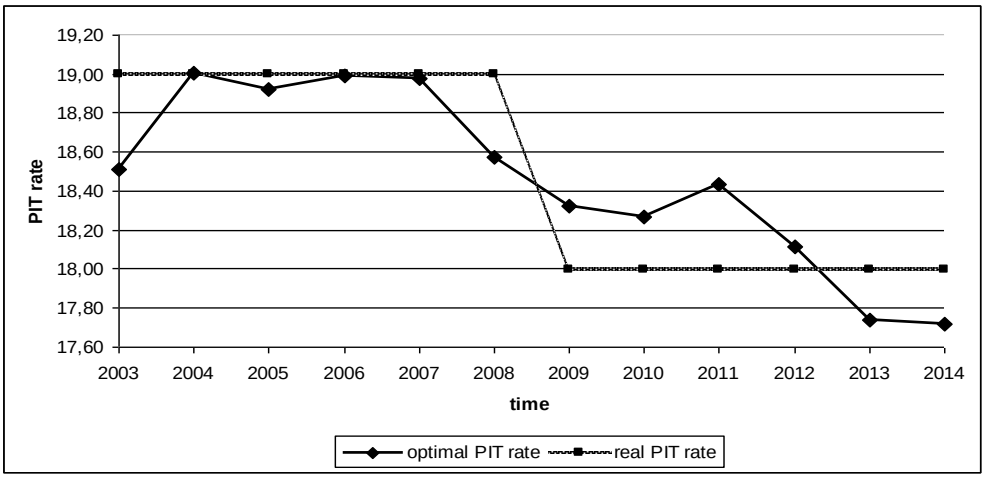


**Fig. 1.** Optimal and real basic PIT rate.

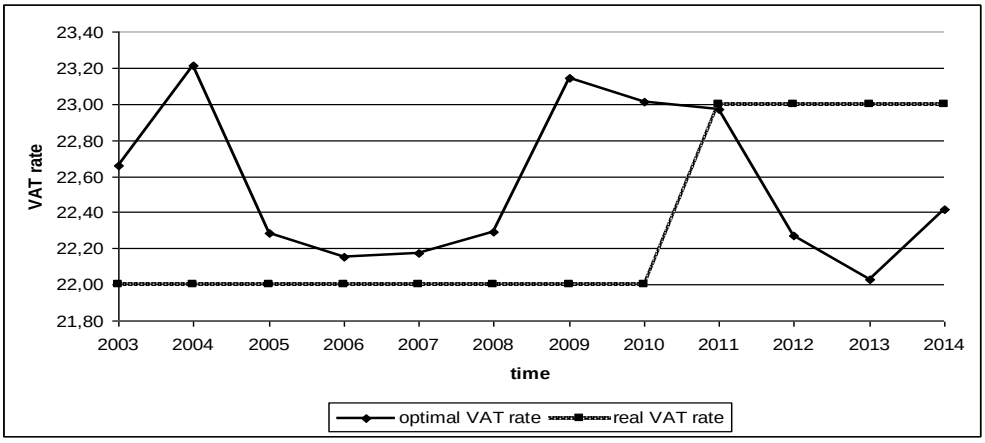


**Fig. 2.** Optimal and real basic VAT rate

The Table 2 presents the optimal and real values of interest rate.

| Year | The optimal values of interest rate | The real values of interest rate |
|---|---|---|
| 2003 | 5.05 | 5.56 |
| 2004 | 5.35 | 5.81 |
| 2005 | 5.16 | 5.25 |
| 2006 | 4.11 | 4.06 |
| 2007 | 4.31 | 4.48 |
| 2008 | 5.14 | 5.73 |
| 2009 | 3.91 | 3.67 |
| 2010 | 3.57 | 3.50 |
| 2011 | 3.97 | 4.25 |

| | | |
|---|---|---|
| 2012 | 4.64 | 4.60 |
| 2013 | 3.70 | 2.92 |
| 2014 | 2.61 | 2.38 |

**Table 2.** The optimal and the real values of interest rate.

The obtained results we present also in the figure below.

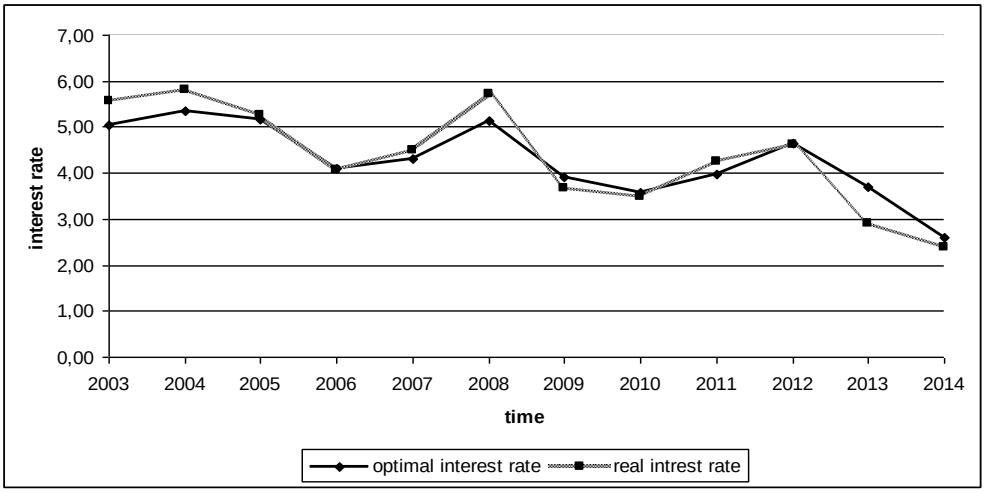


**Fig. 3.** Optimal and real values of interest rate

The application of the optimal values of tax rates and the interest rate in the conduct of fiscal and monetary policy allows to achieve the minimum deviation of state variables from the desired value of these variable, ie inflation from the inflation target, GDP from potential GDP and the general government deficit to GDP ratio from the set value in the convergence criterion.

## Concluding remarks

In the article we determined the fiscal policy and monetary policy rules that are the feedback rules. They are the solution of the Quadratic Linear Tracking Problem. We calculated the optimal values of tax rates: the basic PIT rate and the basic VAT rate and also we calculated the optimal values of interest rate. As the economy can be regarded as a dynamic system with control, thus application of the solution of Quadratic Linear Problem will help the economy develop in accordance with the desired path. The determined optimal fiscal and monetary policy rules have a positive impact on economy because they minimize the deviation of inflation, GDP and government deficit from the desired values. In the simple proposed

optimal fiscal and monetary policy rules, the tax rates and the interest rate depends on the inflation rate, the GDP growth rate and the general government deficit to GDP ratio. However we made the analysis ex post. When we know the predicted values of the state variables: the inflation rate and GDP growth rate and also the forecasts of the general government deficit to GDP ratio we can calculated the fiscal and monetary policy rules and we can calculate the optimal values of forecast of tax rates and interest rate. For the analysis ex ante we must use the forecasts that can be determined different methods. Selected methods of calculation of forecasts discusses Hadaś-Dyduch (see Hadaś-Dyduch, 2015, 2016).

**References**

Benigno, P., & Woodford, M. (2012). Linear-quadratic approximation of optimal policy problems. *Journal of Economic Theory, 147*(1), 1-42.

Ellison, M. (n.d.). *Optimal Linear Quadratic Control*. Lecture. Retrieved from http://users.ox.ac.uk/~exet2581/recursive/lqg_mat.pdf.

Ferrante, A., & Ntogramatzidis, L. (2013). The generalised discrete algebraic Riccati equation in linear-quadratic optimal control. *Automatica, 49*(2), 471-478.

Hadaś-Dyduch, M. (2014). The market for structured products in the context of inflation. In M. Papież & S. Śmiech (Eds.), *Proceedings of the 8th Professor Aleksander Zelias International Conference on Modelling and Forecasting of Socio-Economic Phenomena*. Cracow: Foundation of the Cracow University of Economics, 47-56.

Hadaś-Dyduch, M. (2015). Prediction of wavelets analysis. In *Financial management of Firms and Financial Institutions, Proceedings (Part I.) 10th International Scientific Conference*. Ostrava, Czech Republic: VSB-Technical University of Ostrava, Faculty of Economics, Department of Finance, 341-348.

Hadaś-Dyduch, M. (2016). Alignment Wavelets as Main Instruments in the Short-time Term Prediction. In P. Jedlička (Ed.), *Double-blind peer reviewed proceedings of the international scientific conference Hradec Economic Days 2016*. Hradec Králové, Czech Republic: University of Hradec Králové, Executive department: Faculty of Informatics and Management Department of Economics and Department of Management, 62-68.

Kendrick, D. (1982). Stochastic Control for Economic Models. *Journal of Economic Dynamic and Control, 1*(3), 311-313.

Kendrick, D., & Amman, H. (2011). *A Taylor Rule for Fiscal Policy*. Utrecht, Netherlands: Utrecht School of Economics, Tjalling C. Koopmans Research Institute.

Przybylska-Mazur, A. (2013). Selected methods of choice of the optimal monetary policy transmission horizon. In M. Papież & S. Śmiech (Eds.), *Proceedings of the 7th Professor Aleksander Zeliaś International Conference on Modelling and Forecasting of Socio-Economic Phrnomena*. Cracow: Foundation of the Cracow University of Economics, 133-140).

Svensson, L. E. (1998). *Inflation targeting as a monetary policy rule*. Cambridge, MA: National Bureau of Economic Research.