

A comparison of alternative proxies for the Fisher price index

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Abstract

The Consumer Price Index (CPI) is used as a basic measure of inflation. The index approximates changes of costs of households' consumption that provide the constant utility (COLI, Cost of Living Index). In practice, we use the Laspeyres price index in the CPI measurement and it may lead to the CPI substitution bias. In the paper we present and compare several price indices being proxies for the Fisher index and thus reducing the above-mentioned CPI bias. Our main conclusion for increasing prices case is that the best approximation of the Fisher price index is obtained for the Young index or the geometric Lowe index.

Keywords: CPI, COLI, the Laspeyres index

JEL Classification: E1, E2, E3

1. Introduction

The *Consumer Price Index* (CPI) is commonly used as a basic measure of inflation. The index approximates changes in the costs of household consumption assuming the constant utility (COLI, *Cost of Living Index*). In practice, the Laspeyres price index is used to measure the CPI (White, 1999; Clements and Izan, 1987). The Laspeyres formula does not take into account changes in the structure of consumption, which occur as a result of price changes in the given time interval. It leads to the conclusion that the Laspeyres index can be biased due to the commodity substitution. Many economists consider the *superlative indices* (like the Fisher index or the Törnqvist index) to be the best approximation of COLI (Lippe, 2011). The difference between the Laspeyres index and the superlative index should approximate the value of the commodity substitution bias. However there are some ways to reduce that bias, like using the Lloyd–Moulton price index – see (Lloyd, 1975; Shapiro and Wilcox, 1997; Białek, 2014, 2015), the AG Mean index (Lent and Dorfman, 2009) or Lowe and Young indices (Armknetch and Silver, 2012). In this paper we examine the effectiveness of these methods in a simulation study. In particular, we intend to approximate the *ideal* Fisher price index by using all the above-mentioned formulas.

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2. The Fisher price index in CPI substitution bias calculations

Let $E(P, \bar{u}) = \min_Q \{P^T Q \mid U(Q) \geq \bar{u}\}$ be the expenditure function of a representative consumer which is dual to the utility function $U(Q)$. In other words it is the minimum expenditure necessary to achieve a reference level of utility \bar{u} at vector of prices P . Then the Konüs cost of living price index is defined as:

$$P_K = \frac{E(P^t, \bar{u})}{E(P^s, \bar{u})} \quad (1)$$

where t denotes the current period, s denotes the base period, and in general, the vector of N considered prices at any moment τ is given by $P^\tau = [p_1^\tau, p_2^\tau, \dots, p_N^\tau]^T$. P_K is a true cost of living index in which the commodity Q changes as the vector of prices facing the consumer changes. The CPI, in contrast, measures the change in the cost of purchasing a fixed basket of goods at a fixed sample of outlets over a time interval, i.e. $Q^s = [q_1^s, q_2^s, \dots, q_N^s]^T = Q^t$. The CPI is a Laspeyres-type index defined by:

$$P_{La} = \frac{\sum_{i=1}^N q_i^s p_i^t}{\sum_{i=1}^N q_i^s p_i^s}, \quad (2)$$

so we assume here the constant consumption vector on the base period level. It can be shown (Diewert, 1993) that under the assumption that the observed period t consumption vector Q^t solves the period t expenditure minimization problem, then:

$$P_K = \frac{E(P^t, U(Q^s))}{E(P^s, U(Q^s))} \leq P_{La}, \quad (3)$$

so $P_{La} - P_K$ is the extent of the commodity substitution bias, where P_K plays the role of the reference benchmark. In the so called economic price index approach many authors use superlative price indices to approximate the P_K index (White, 1999). Thus in general, we can use any superlative price index P_{sup} to calculate the above mentioned CPI bias, namely:

$$B_{csub} \approx P_{La} - P_{sup}. \quad (4)$$

In particular, many authors use the superlative Fisher index to calculate B_{csub} (White, 1999) and obtain:

$$B_{csub} \approx P_{La} - P_F. \quad (5)$$

We can evaluate both COLI and CPI substitution bias (5) using the Fisher index only after observations of Q^t . It is not convenient in practice because we need to use current prices to calculate the Paasche price index and next the Fisher formula. It would be ideal to approximate the Fisher price index using only base-period expenditure data (or older). This is the main aim of our work.

3. Approximations of the Fisher index – alternative price indices

3.1. The Lloyd–Moulton price index

A superlative Fisher price index can be approximated by using the Lloyd–Moulton price index (Lloyd, 1975; Shapiro and Wilcox, 1997). It does not make use of current-period expenditure data, so it is even possible to approximate a superlative index in real time and extrapolate the time series. The Lloyd–Moulton price index formula is as follows:

$$P_{LM}(\sigma) = \left[\sum_{i=1}^N w_i^s \left(\frac{P_i^t}{P_i^s} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (6)$$

where σ is some real parameter and w_i^s denotes the expenditure share of commodity i in the base period s . We can also find some modifications of P_{LM} in the literature (Białek, 2015). Empirical studies on the proper value of σ can be found in the following papers: (Feenstra and Reinsdorf, 2007; Biggeri and Ferrari, 2010; Greenlees, 2011; Armknecht and Silver, 2012).

3.2. The Young Price index

The CPI is calculated as a weighted arithmetic mean of price relatives, where the weights are the expenditure shares in period s . In practice there is a lag between the expenditures share survey period (τ) and their first use in the index because the compilation of the household expenditure data needs time. Statistical agencies use a prior period τ survey weights to rebase a CPI that runs from the price reference period s , i.e. $\tau < s < t$. As a result it can be proposed the Young price index:

$$P_Y = \sum_{i=1}^N w_i^\tau \left(\frac{P_i^t}{P_i^s} \right) \quad (7)$$

where

$$w_i^\tau = \frac{P_i^\tau q_i^\tau}{\sum_{k=1}^N P_k^\tau q_k^\tau} \quad (8)$$

We consider also the geometric Young price index, which is given by:

$$P_{GY} = \prod_{i=1}^N \left(\frac{P_i^t}{P_i^s} \right)^{w_i^t} . \quad (9)$$

3.3. The Lowe Price index

In the paper of (Armknrecht and Silver, 2012) we can also find the so called Lowe price index formula, which can be expressed as follows:

$$P_{Lo} = \frac{\sum_{i=1}^N P_i^t q_i^\tau}{\sum_{i=1}^N P_i^s q_i^\tau} = \sum_{i=1}^N w_i^{\tau,s} \left(\frac{P_i^t}{P_i^s} \right) \quad (10)$$

where

$$w_i^{\tau,s} = \frac{P_i^s q_i^\tau}{\sum_{k=1}^N P_k^s q_k^\tau} . \quad (11)$$

In the cited paper we can read: “More typically, weights are price-updated between period τ and the price reference period s to effect fixed period- τ quantities”. Similarly to (9) we can also use the geometric version of the Lowe price index, namely:

$$P_{GLo} = \prod_{i=1}^N \left(\frac{P_i^t}{P_i^s} \right)^{w_i^{\tau,s}} . \quad (12)$$

4. Simulation

Empirical studies on the Lloyd-Moutlon price index can be found in papers: (Greenlees, 2011; Armknrecht and Silver, 2012; Białek, 2014). In this paper we intend to examine the usefulness of the rest of mentioned indices. Let us take into consideration a group of $N = 6$ components, where prices and quantities change linearly² between two fixed time moments³ $s = 0$ and $t = 1$, namely:

$$p_i^T = p_i^0 + (p_i^1 - p_i^0)T , \quad (13)$$

$$q_i^T = q_i^0 + (q_i^1 - q_i^0)T \quad (14)$$

² It is typical for the analytical purposes to assume a linear or exponential change in quantities and prices (see for instance Vaninsky, 2014).

³ The considered time interval $[0, 1]$ is taken for convenience. In practice, any time interval (month, year, 10 years) can play that role.

where $i \in \{1, 2, \dots, 6\}$, $T \in [0,1]$ and p_i^0 , p_i^1 , q_i^0 and q_i^1 are fixed real numbers (see Table 1). Hence: $p_i^{T=0} = p_i^0$, $p_i^{T=1} = p_i^1$, $q_i^{T=0} = q_i^0$ and $q_i^{T=1} = q_i^1$. We consider here only one case where most of prices increase (some other cases will be presented in the extended version of the paper).

Good	Prices and quantities			
$i =$	p_i^0	p_i^1	q_i^0	q_i^1
1	100	110	1,050	900
2	11	12	10,000	8,000
3	50	62	25	20
4	100	105	2,020	1,950
5	200	150	300	185
6	75	96	2,000	1,800

Table 1. Value of prices and quantities for six commodities.

We consider several values of τ for comparisons between price indices described in the paper, i.e. $\tau \in \{-1, -0.75, -0.5, -0.25\}$. The values of all considered price indices and their distances to the Fisher price index are presented in Table 2. The smallest distances to the Fisher price index presented in tables are in bold.

Formula	$\tau = -1$	$\tau = -0.75$	$\tau = -0.5$	$\tau = -0.25$
P_{La}	1.0947	1.0947	1.0947	1.0947
P_{Pa}	1.0801	1.0801	1.0801	1.0801
P_F	1.0874	1.0874	1.0874	1.0874
P_Y	1.0887	1.0901	1.0916	1.0931
P_{GY}	1.0795	1.0806	1.0817	1.0883
P_{Lo}	1.1074	1.1044	1.1013	1.0981
P_{GLo}	1.0992	1.0957	1.0921	1.0883

$P_{La} - P_F$	0.0073	0.0073	0.0073	0.0073
$P_Y - P_F$	0.0013	0.0027	0.0047	0.0057
$P_{GY} - P_F$	-0.0078	-0.0068	-0.0056	-0.0043
$P_{Lo} - P_F$	0.0200	0.0169	0.0138	0.0106
$P_{GLo} - P_F$	0.0117	0.0082	0.0046	0.0009

Table 2. The values of the considered price indices and their distances to the Fisher index.

Conclusions

In the considered situation when most of prices increase (see Simulation Study, Table 1), we obtained the best approximation of the Fisher price index for the Young index or the geometric Lowe index. The geometric Lowe index seems to be a better Fisher index approximation for small values of τ parameter. This is a good information since the Lowe formula is used by US or many other countries for CPI compilation (in Poland CPI is calculated by using the Laspeyres formula). Let us notice that the CPI substitution bias, calculated as in (5), is 0.73 *p.p.* and it is not negligible. This kind of CPI bias can be relatively big and, similarly to our results, as a rule it has a positive value (Woolford, 1994; Hoffmann, 1999; Filer and Hanousek, 2003; Schultze and Mackie, 2002; Frenger, 2006). It is very important to reduce it and this reduction is quite effective when we use the presented, alternative indices. Let us also notice that the discussed approximations of the Fisher index alleviate both the downward bias (Geometric Young index) and upward bias (Geometric Lowe and Young indices). Similar conclusion drawn for the Törnqvist price index (also superlative) can be found in (Armknrecht and Silver, 2012).

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