Lowering costs of private retirement by sharing of life-length risk within a household

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Abstract

In this article there is used a discrete-time, cash-flow based, two-person household financial plan optimization model, presented earlier by Feldman, Pietrzyk and Rokita (2014a) and Pietrzyk and Rokita (2015b). It is shown by an example that the model captures internal transfer of life-length risk within a household (sharing risk of longevity and premature death between household members) and that it allows to reflect advantages from this effect in terms of retirement investment contribution reduction.

Keywords: *household, financial planning, retirement investment, longevity risk, risk sharing*

JEL Classification: D91, E21, J26, D10

1. Introduction

As it was observed many times in retirement planning research (Kotlikoff and Spivak, 1981; Brown and Poterba, 2000; Brown, 2001) – with reservation by Hayashi et al. (1996) – household members share their longevity and premature-death risk, and this phenomenon is reflected in their long-term financial decisions. This is why any financial planning model for households should allow for this property. Feldman et al. (2014a) and Pietrzyk and Rokita (2015b) proposed a model, in which an intuitive and easily applicable way of expressing life-length risk aversion was introduced, and, moreover, financial plan optimization problem was naturally fitted to the risk aversion. At the same time, this gave an additional advantage, consisting in a substantial reduction of the number of survival scenarios considered. This advantage would, however, not be sufficient if the model did not allow to capture risk sharing. Pietrzyk and Rokita (2016) showed that there is a life-length-risk sharing effect in the model. The aim of this research is, in turn, checking if it is reflected in the contributions that households need to invest in private pension plans.

The development of classical life-cycle consumption optimization models from their univariate (single person) version, on the pattern of Yaari (1965), to household generalization

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was started by Kotlikoff and Spivak (1981). A formal model for a married couple was then proposed by Hurd (1999). Brown and Poterba (2000) analyzed joint annuities suited to married couples and examined their advantages.

Here in this paper, no joint financial instruments are considered. This is not just a simplification, but it enables comparability with a benchmark of two separate persons.

Comparison with the benchmark of two persons treated separately opens a new direction of applications of the model. Originally it was proposed as a financial plan optimization supporting tool. Now, an attempt is made to use it as a tool for measuring the potential of economization on pension plan investment, obtained as a result of household internal risk and capital transfers, without giving up life standard in retirement. The outcome may be just the difference between required contributions for two separate persons and a couple.

2. General characteristic of the model

The research is based on the model described by Pietrzyk and Rokita (2015b, 2016). It is a discrete-time, cashflow-based model of life-long financial plan for a two-person household. Its simplified version, with only two financial goals, namely – retirement and bequest, is used here. This is sufficient if sharing of life-length risk and its effect on retirement investment contributions is a focus of attention. But there have already been presented also multiple-goal variants (Feldman et al., 2014b; Pietrzyk and Rokita, 2014a, 2014b).

The concept is that that a retirement-goal-oriented skeleton is constructed first and then it may be augmented in many directions. This is justified by a special role of retirement planning in personal finance (high magnitude and no possibility of postfinancing).

In the model by Feldman et al. (2014a) and Pietrzyk and Rokita (2015b) it is assumed that, at the starting point, the household divides all its income into two parts only: consumption and retirement investment. It is also assumed that the household maintains a constant proportion in which pension plans of each person are contributed to. There are two decision variables in the financial plan optimization problem:

- proportion of initial investment-consumption division,
- proportion in which retirement investment is assigned to pension plans of the first and the second person.

It is assumed that private retirement investment of a given person is liquidated on retirement date of this person and used for purchasing a life annuity for this person.

While retirement goal is defined by in terms of time and magnitude (typical way of setting financial goals), the bequest goal is set in a less strict way. The household declares just a bequest motive parameter, which may be also interpreted as a reverse of propensity to consume.

Under assumption of the model, consumption needs and the labor income of the household fallow deterministic patterns (e.g., constant growth rates). At the moment zero there is no surplus generated (income fully spent on consumption and investment). In any subsequent period, a surplus or a shortfall may occur (if income and consumption dynamics differ).

In the model there are only two types of investment:

- a defined contribution private pension plan,
- risk free, high liquidity investment of the surplus.

The first (pension plan investment) is assumed to be a risky investment. All asset allocation decisions for it are assumed to have already been made before the start of the plan.

As far as the second type of investment is considered, investing of the surplus is not planned, actually. However, if a surplus is generated, it should not be left uninvested. On the other hand, it cannot be assumed that the surplus is invested at a high rate. This is because of the nature of this financial category. The surplus is a side effect rather than a goal. Moreover, cumulated surplus plays a role of high liquidity reserve, that is used on a contingent basis. This means that a realistic plan must assume rather a low rate of return on invested cumulated surplus (e.g., zero percent in real terms).

There are two risk factors in the basic version of the model – dates of death of the two household members. Each pair of particular realizations of these random variables will be further referred to as a *survival scenario*. The survival scenarios may be modeled as realization of some survival processes. For the numerical example presented in this article survival probabilities are calculated on the basis of two independent Gompertz (1825) models.

The approach used here is elastic in many respects. For example, it does not impose any particular model of the survival process. There is also no limitation as to the choice of the model describing consumption and income dynamics. The intention was to construct a modular framework independent of technical details of its particular elements.

3. Financial plan optimization

The goal function of the optimization procedure is composed of expected discounted utility of consumption and bequest. It will be further called value function. The optimization procedure is constructed on a greed of a discrete number of survival scenarios (pair of dates of death).

Not the whole range of possible dates of death is covered by the optimization procedure. An important concept of the proposed approach is to adjust the choice of survival scenarios to the household aversion against life-length risk. It is assumed that the more risk averse the household, the more scenarios (and the less probable ones) should be taken into account when optimizing the financial plan. The household with higher risk aversion will optimize its financial plan for scenarios that deviate from the expected one to a higher extend. Thus, the more risk averse the decision maker, the broader range of considered scenarios. It is assumed that the household is able to declare the width of the range of scenarios in terms of the number of years before and after expected dates of death. This range is called here the *range of concern* and its defined by eq. (1) – compare, for example, (Pietrzyk and Rokita, 2015b):

$$G_H^* = \left[E(D_1) - \gamma^*; E(D_1) + \delta^* \right] \times \left[E(D_2) - \gamma^*; E(D_2) + \delta^* \right] \quad (1)$$

where γ^* – premature-death risk aversion parameter (number of years before expected time of death that household takes into consideration), δ^* – longevity risk aversion parameter (years after expected death), $E(D_i)$ – unconditional expected time of death of Person i (i.e., $E(D_i) \equiv E(D_i \mid D_i > t_0)$), t_0 being the starting moment of the plan.

The value function of the household is defined by eq. (2). The optimization consists in maximization of the value function with aforementioned decision variables, under minimum consumption constraint and budget constraint. The double summation loop stands for taking survival scenarios from within the range of concern. For each scenario unconditional probability (of the whole scenario) is taken. Discounted utility of consumption are summed through the whole trajectory of a given scenario. To that, discounted utility of bequest from the end of each scenario is added:

$$V(c_0, v) = \sum_{D_2^* = E(D_2) - \gamma^*}^{E(D_2) + \delta^*} \sum_{D_1^* = E(D_1) - \gamma^*}^{E(D_1) + \delta^*} p_{D_1, D_2} \left[\begin{array}{c} \alpha \left(\sum_{t=0}^{\max\{D_1^*, D_2^*\}} \frac{1}{(1+r_c)^t} u\left(C\left(t; D_1^*, D_2^*\right)\right)\left(\gamma(t) + \delta(t)\right) \right) + \\ \beta \frac{1}{(1+r_B)^{\max\{D_1^*; D_1^*\}}} u\left(B\left(\max\{D_1^*, D_2^*\}; D_1^*, D_2^*\right)\right) \end{array} \right] \quad (2)$$

$$\rightarrow max$$

where $u(.)$ – utility function (the same in all segments of the formula); c_0 – consumption rate at the moment 0; v – proportion of Person 1 investment in total one-period contribution of the household in the first period $(v \equiv v_1, v_1 = 1 - v_2)$; $\gamma(t)$ – premature death risk aversion measure (depends on γ^*); $\delta(t)$ – longevity risk aversion measure (depends on δ^*); p_{D_1, D_2} – probability of such scenario that $(D_1 = D_1^*, D_2 = D_2^*)$; α – consumption preference parameter; β – bequest preference parameter; $\max\{D_1^*, D_2^*\}$ – time of household end under

$\left(D_1^*, D_2^*\right)$ scenario; $C\left(t; D_1^*, D_2^*\right)$ – consumption at the moment t under $\left(D_1^*, D_2^*\right)$ scenario; $B\left(t; D_1^*, D_2^*\right)$ – cumulated surplus at the moment t under $\left(D_1^*, D_2^*\right)$ scenario for $t = max\left\{D_1^*, D_2^*\right\}$ this is just amount of available bequest); r_C – discount rate of consumption; r_B – discount rate of bequest.

Multipliers $\gamma\left(t\right)$ and $\delta\left(t\right)$ are functions of time and depend on parameters γ^* and δ^* which is explained in more details in (Feldman et al., 2014b).

4. Risk sharing effect

The result of financial planning for a given survival scenario may be presented graphically in a form of a cumulated surplus trajectory plot. Albeit main financial categories used in the model are cashflows, rather than financial resources (wealth), the cumulated surplus trajectory well reflects financial situation of the household. Also its vulnerability to deviations from expected scenario may be easy illustrated by plotting cumulated surplus trajectory under different scenarios. Moreover, the very shape of cumulated surplus trajectory for the expected scenario is often sufficient to provide an analyst with a rough idea of financial plan riskiness.

The analysis of life-length-risk sharing effect is based here on comparison of cumulated surplus trajectories for two household variants: joint and disjoint. The joint variant corresponds to the household as described in section 2. The optimization is made for the whole household. The disjoint variant is constructed on the basis of the joint (household) one, in the way described by Pietrzyk and Rokita (2016). From the research presented there it may be concluded that the financial plan for a household (joint variant) is much less sensitive to influence of unexpectedly early death and unexpected longevity. The disjoint variant never performs better than the joint one if other assumption of the model hold. By better performance, a higher reading of the value function, as described by eq. (2), is understood. In financial terms it means that for the disjoint variant a shortfall is incurred in a bigger number of scenarios. And, if there is a shortfall also for the joint variant, in the disjoint variant the shortfall is deeper under the same survival scenario.

Lower life-length risk for a joint variant than in the disjoint one, other things being equal, is a result of risk sharing in a household. The mechanism of this risk sharing is based on internal financial transfers between household members. In the model there are four channels of the transfer:

1. Common spending of incomes for consumption.
2. Investing for individual pension plans from a common pool of financial means.
3. unlimited access to common cumulated surplus.
4. Possibility of inheriting (some part) cumulated retirement investment of one person by the second household member in case the firs dies before retirement.

5. Reduction of investment contribution

The differences in performance between joint and disjoint variant observed by Pietrzyk and Rokita (2015a, 2016) may be also analyzed from the perspective of investment contribution. Let, in a particular scenario, the disjoint variant end with lower cumulated surplus than the joint one. This is a typical situation. One may ask then, how to modify the financial plan for the disjoint variant to force its final residual wealth to be equal to the residual wealth of the joint variant (under the same scenario). This operation may be performed only for each particular scenario separately, because in each scenario the residual wealth is different. The answer may be increasing investment contributions in the disjoint variant. In an adjusted disjoint variant, obtained in this way, the sum of investment contributions from the two persons is higher than the corresponding sum invested by a household.

This property may be illustrated by the following numerical example. The household members are a 36 year old man and 31 year old woman, with incomes of 76,000 and 52,000 monetary units, respectively. Life expectancies are 74 and 82 (age). Joint annual consumption of the household (after optimization of the plan for the joint variant) is 100,880, which includes 50,000 of common consumption (not attributed to any particular person), 23,127 assigned to the woman, and 27,753 assigned to the man. Retirement age is equal for men and women and it is 67. Replacement rate forecast for the woman is 35% and for the man it is 40%. Household consumption is assumed to grow in real terms at the rates of 1.6% – for common and 2% – for individually assigned. Income growth rate is set at 2%. This allows to generate surpluses form period to period. Long-term average return on investments is set at 2% annually, in real terms.

The household has declared risk aversion parameters of: $\gamma^* = 5$ and $\delta^* = 5$ (comp. eq. (1)), and bequest motive parameter $\beta = 0{,}25$.

Let *Person 1* be the woman and *Person 2* – the man. Consistently, D_1 denotes date of death of the woman and D_2 – of the man. The example is constructed to illustrate behavior of financial plan for the joint and disjoint variant in two scenarios.

The first scenario is the expected one ($D_1 = E(D_1)$, $D_2 = E(D_2)$), referred further to as *Scenario* (0,0). The symbol (0,0) means that the dates of death deviate by zero years from expected life times.

The second is a scenario from an edge of the range of concern, but still belonging to this range, namely the scenario defined as: ($D_1 = E(D_1) - 5$, $D_2 = E(D_2) + 5$). The scenario is further called *Scenario* (-5,5), which should be read: Person 1 dies five years earlier than her expected life time and Person 2 lives five years longer than his expected life time.

The last scenario is an extremely disadvantageous corner of the range of concern, because the person who is expected to live longer dies as early as the range of concern allows for (realization of premature-death risk), and the person who is expected to live shorter lives to the latest age spanned by the range (realization of longevity risk).

Analysis of further scenarios, falling beyond the range of concern, would be possible but not very informative – performance of the plan under such scenarios would be poor for both variants, which should be no surprise, as the plan optimization procedure neglects these scenarios.

In the Fig. 1, there are presented cumulated surplus trajectories for the joint and disjoint variant, under the scenarios described above, with enforced equal residual wealth. The bar plot represents cumulated surplus in the joint (household) variant and the line stands for sum of cumulated surpluses of the two persons in the disjoint variant. The plots show the idea of equal ends. What is of the main interest here, however, is the difference in investment contributions, which can not be read from the plots. More detailed information is then needed.

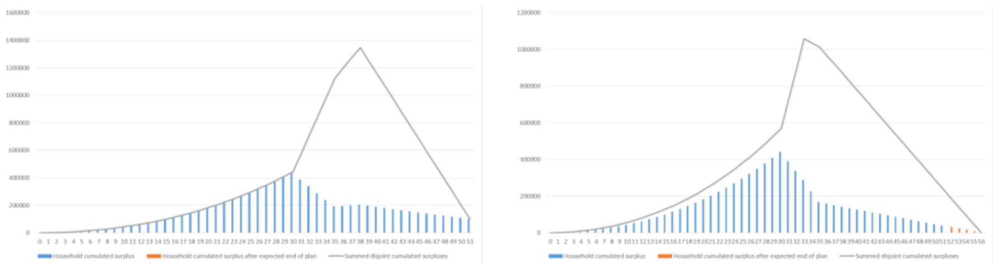

Fig. 1. Optimized financial plan (household vs. disjoint variant) assuming risk aversion parameters of: $\delta^* = 5$ and $\gamma^* = 5$, with enforced equal cumulated surplus levels at the end. Under expected survival times (left) and for Scenario "(-5,5)" (right).

The information on investment contribution may be read from Table 1. In the columns 2-8 of the table main characteristics of financial plan are presented. These columns are scenario-independent. Their interpretation is the following: column 2 ($VC_0^{(1)}$) – consumption assigned to Person 1, column 3 ($VC_0^{(2)}$) – consumption assigned to Person 2, column 4 ($Iv_0^{(1)}$) – investment of Person 1, column 5 ($Iv_0^{(2)}$) – investment of Person 2, column 6 – sum of Person 1 and Person 2 investments, column 7 ($Sp_0^{(1)}$) – surplus at the beginning, assigned to Person 1 (has a clear interpretation only in disjoint variants), column 8 ($Sp_0^{(2)}$) – surplus at the beginning, assigned to Person 2 (as above).

Columns 9 and 10, in turn, show final cumulated surplus, which, of course, depends not only on the choice of the plan, but also on the realized scenario. In the column 9 there is shown cumulated surplus at the end of the expected scenario – Scenario (0,0). In the column 10 cumulated surplus at the end of the Scenario (-5,5) is provided.

Rows of Table 1 correspond to different financial plans. Row 1 contains information about a plan for the household (joint variant) after optimization. Row 2 – informs about a disjoint variant derived from the joint one. Rows 3 and 4 show the results of enforcing in the disjoint variant the same final residual wealth as in the joint variant, under the two aforementioned scenarios. As it can be read from column 6 of the table, enforcement of final cumulated surplus makes it necessary to increase investment contributions. This causes also a decrease of consumption as compared with the joint (household) variant.

	Variant	$VC_0^{(1)}$	$VC_0^{(2)}$	$Iv_0^{(1)}$	$Iv_0^{(2)}$	$Iv_0^{(1)} + Iv_0^{(2)}$	$Sp_0^{(1)}$	$Sp_0^{(2)}$	$CSp_{\max(D_1^*, D_2^*)}$ **scenario (0,0)**	$CSp_{\max(D_1^*, D_2^*)}$ **scenario (–5,5)**
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
1	household variant	23,127	27,752	0	27,120	**27,120**	0	0	101,585	305
2	disjoint variant	23,127	27,752	27,873	0	27,873	0	-753	59,728	-1,356,410
3	adjusted disjoint variant (adjusted final effect at $E(D)$)	24,583	26,166	26,417	834	27,251	0	0	101,585	-----

4	adjusted disjoint variant (adjusted final effect at D_1= $E(D_1)$–5, D_2= $E(D_2)$+5	22,085	17,517	28,915	9,483	38,398	0	0	-----	305

Table 1. Consumption (*VC*), investment (*Iv*) and cumulated surplus (*CSp*) of two persons in an optimized financial plan with risk aversion (γ*=5, δ*=5) (household vs. disjoint variant).

Conclusion

The results presented by Pietrzyk and Rokita (2016) and here in this paper are examples supporting the conjecture that the two-person household financial plan model with optimization adjusted to life-length risk aversion, in the form presented by Feldman et al. (2014a) and Pietrzyk and Rokita (2015b), is fit for reflecting household effect (understood as advantages of capital transfer and risk sharing between household members) in respect of life-length risk. As the household effect is, at least since the research by Kotlikoff and Spivak (1981), a commonly known and very important property in personal finance, an acceptable household financial planning model must allow for this effect. Here, it is shown that the discussed model does not stand in contradiction with the common intuition and knowledge. Moreover, it has been illustrated by an example that the model may be used to identify a potential for economizing on private pension investment contributions. This may, in turn, also be used to encourage households to start saving and investing for their retirement, by showing that it may be less demanding if household effect is taken into account.

Further research in this area may entail development of a risk measure for a long-term household financial plan. Then, the relationship between risk sharing and overall risk of the plan might be more formally analyzed.

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