

Detecting structural changes in the time series in the use of simulation methods

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Abstract

This paper proposes a test to detect structural changes in the time series. The proposed procedure uses simulation methods, which specificity allows to take the very general assumptions. In particular, the proposed method can be used to detect the level of breakpoints in analyzed phenomenon. The proposed solution was compared to the solutions known from the literature using computer simulation.

Keywords: structural break, permutation test, Chow test, Nyblom-Hansen test, CUSUM

JEL Classification: C120, C140, C150

1. Introduction and basic denotations

In practice of economic research it is often to be faced with a situation in which it is necessary to detect structural changes of analyzed phenomenon. Structural changes are one of the most important issues in the analysis of non-stationary time series (Pang and Ting, 2005a). In the literature, one can find many solutions to detect such changes. Several tests to check stationary of time series are presented in Domański et al. (2014). One of the basic statistical tests is the test proposed by Chow (1960) and Nyblom-Hansen (Hansen, 1992; Nyblom, 1989). It involves comparing two linear regression models, on the basis of separate groups of data. Another approach presents a method of cumulative sums of squares: CUSUM proposed by Brown, Durbin and Evans (1970). It is one of the best known and developed methods for the detection of changes in the structure of the time series. This method went through many modifications. Ting Pang developed a centered cumulative sums of squares method and developed it by examining the median deviation SUMSRM (Pang and Ting, 2005b). To detect changes also used the method of maximum likelihood ratio and many modifications based on F statistics (Zeileis, 2005).

Ignoring the presence of structural change in time series analysis, carries a certain consequences which are manifested in the reduction of its accuracy. Timely detection of structural changes in the prices of listed shares in the financial markets, exports and GDP size, allows to achieve benefits for the company, it is therefore very important in terms of time.

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This is also significant in the analysis of production process to detect the appearance of trend. It could be the signal which may indicate a deregulated production process.

The proposed solution is free from the basic assumptions on the time series, which is very common in the use of other methods. For example, in the case of the Chow test, it is assumed that the random component of the estimated model is independent and normally distributed with unknown variance.

A stochastic process can be determined by means of symbols $\{Y_t, t = 1, 2, \dots\}$, using symbols y_1, y_2, \dots, y_n the elements of the time series can be marked, which are the execution of the process. The main purpose is to detect changes in the point k , where $1 < k < n$. Examples of changes are schematically shown in Figure 1. We can talk about the change in the mean value of the analyzed process, the appearance of a trend or combination of both situations in different ways.

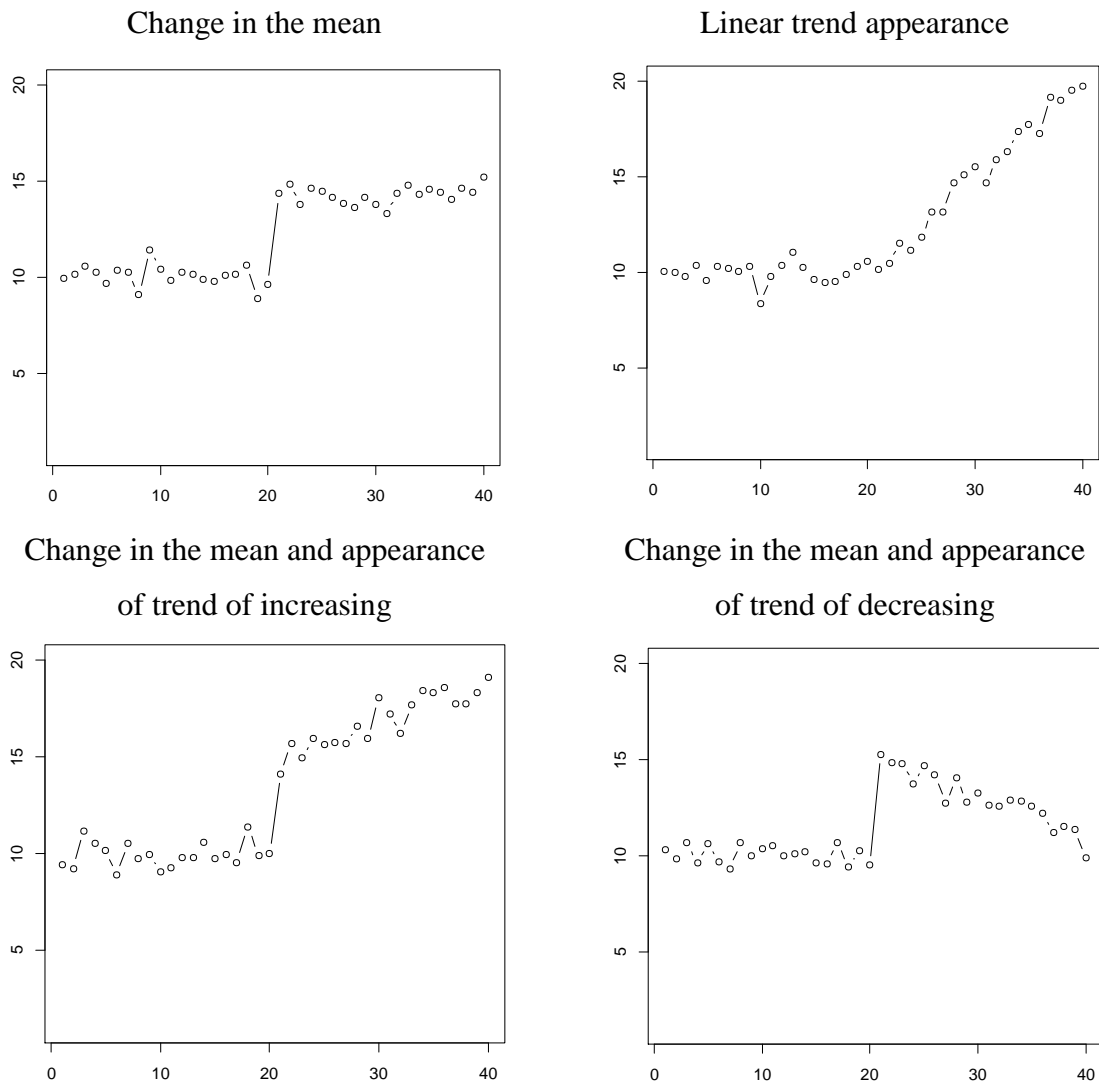


Fig. 1. Trend change examples.

2. Permutation tests

Permutation tests are one of the methods of computer-aided simulation. Tests were introduced by R. A. Fisher and E. J. G. Pitman in the thirties of the twentieth century (Kończak, 2012). In these tests, the observed value of the test statistic is compared with the empirical distribution, under the truth of the null hypothesis. The idea of the test is much simpler than the tests based on the normal distribution. Main area of its application is the classic two-sample test (Efron and Tibshirani, 1993). In order to perform the test procedure for permutation tests, these steps should be followed (Good, 2005):

1. Setting the null hypothesis and the alternative.
2. Determine the form of the test statistics (T).
3. Calculate the value of the test statistics (T_0) on the basis of empirical data.
4. The evaluation of the distribution of the test statistics based on simulations assuming the veracity of H_0 (T_1, T_2, \dots, T_N , where $N > 1000$).
5. Decide on the basis of the resulting distribution of the test statistics.

Decision is made on the basis of the value of *ASL* – Achieving Significance Level (*p*-value equivalent in standard statistical tests).

ASL is expressed as follows:

$$ASL = P(\hat{\theta} \geq \hat{\theta}_0). \quad (1)$$

Is estimated based on empirical data using the following formula:

$$ASL \approx \frac{\text{card}\{i : \hat{\theta}_i \geq \hat{\theta}_0\}}{N}. \quad (2)$$

This notation means that the critical area of the test statistics is right-handed. To get the left-hand area, the appropriate changes in the above inequalities are needed. If $ASL < \alpha$ it is decided to reject H_0 , otherwise, there are no basis to reject this hypothesis.

Parametric tests are normally used for calculating population parameters such as mean, variance and rate structure. These tests, except the last, require that the sample was collected from a normally distributed population. For larger samples limit distributions can be used. In the next part of the study, to detect the appearance of structural change in time series a permutation test will be used. Considered time series will be divided into two subseries as follows: y_1, y_2, \dots, y_k and $y_{k+1}, y_{k+2}, \dots, y_n$.

3. The test procedure

Hypothesis which says that at the point k ($1 < k < n$), there has been a structural change will be verified. Alternative hypothesis is the negation of the null hypothesis. Permutation tests make it possible to design a test statistic in any way. In this paper the test statistic was proposed as follows:

$$T = |b_2 - b_1| + \left| \frac{a_2}{a_1} \right| \quad (3)$$

where

$$a_1 = \frac{\sum_{t=1}^k (t - \bar{t})(y_t - \bar{y})}{\sum_{t=1}^k (t - \bar{t})^2}, \quad a_2 = \frac{\sum_{t=k+1}^n (t - \bar{t})(y_t - \bar{y})}{\sum_{t=k+1}^n (t - \bar{t})^2},$$

$$b_1 = \bar{y} - a_1 \bar{t}, \quad b_2 = \bar{y} - a_2 \bar{t},$$

$$\bar{y} = \frac{1}{k} \sum_{t=1}^k y_t, \quad \bar{y} = \frac{1}{n-k} \sum_{t=k+1}^n y_t,$$

$$\bar{t} = \frac{1}{k} \sum_{t=1}^k t, \quad \bar{t} = \frac{1}{n-k} \sum_{t=k+1}^n t.$$

The test procedure of a permutation test to detect a structural change at time $t = k$ is as follows:

1. Establishment of the level of significance α .
2. Calculate the T_0 value of statistic T based on simulated data.
3. Execution the time series permutations of N times, then calculating the value of the test statistics.
4. On the basis of the empirical distribution of the test statistics T , the ASL value is calculated.

If $ASL < \alpha$, then the hypothesis H_0 is rejected, otherwise there is no basis to reject H_0 .

As the number of repetitions of permutations assumed $N = 1000$.

4. Use the permutation test to detect structural changes

One of the elements of time series analysis is to detect structural changes. Their detection can be crucial for the accuracy of its analysis and estimation of the model. In order to detect structural change, the following methods were used: the proposed method based on a permutation test, the Chow test, the Nyblom-Hansen test and CUSUM. Computer simulation

was carried out based on the following assumptions: the point k process is stable, in step $k + 1$ there is a structural change. For each of the tested methods in the following disorders of varying strength: a change in the mean, the appearance of a linear trend, a change in the mean and the appearance of a linear trend.

The strength of the various disorders were controlled by taking the mean value of offset parameter levels: $\Delta m = 0; 1; 2; 4$, and the slope of the regression line levels: $\Delta a = 0; 0.05; 0.1; 0.2$. At each iteration of the simulation, 20 values were generated from the distribution of stable and 80 in accordance with the disorder. For each parameter combination power determined disorder tests was analyzed. The results of this analysis are shown in Table 1. The experiment allowed for the implementation of a combination of 60 tests. Examples of time series under consideration are shown in Figure 2. Figure 3 shows the histograms of the p -values obtained by simulation.

Δm	Test	Δa			
		0	0.05	0.1	0.2
0	Permutation		0.64	0.85	0.93
	Chow		0.31	0.90	1.00
	Nyblom-Hansen		0.07	0.48	1.00
	CUSUM		0.07	0.48	1.00
1	Permutation	0.12	0.39	0.63	0.83
	Chow	0.73	0.41	0.62	1.00
	Nyblom-Hansen	0.29	0.05	0.04	0.78
	CUSUM	0.29	0.05	0.04	0.78
2	Permutation	0.48	0.46	0.46	0.67
	Chow	1.00	0.99	0.96	1.00
	Nyblom-Hansen	0.95	0.61	0.22	0.37
	CUSUM	0.95	0.61	0.22	0.37
4	Permutation	0.98	0.85	0.60	0.51
	Chow	1.00	1.00	1.00	1.00
	Nyblom-Hansen	1.00	1.00	1.00	0.94
	CUSUM	1.00	1.00	1.00	0.94

Table 1. Power of tests comparison in case of different disturbances.

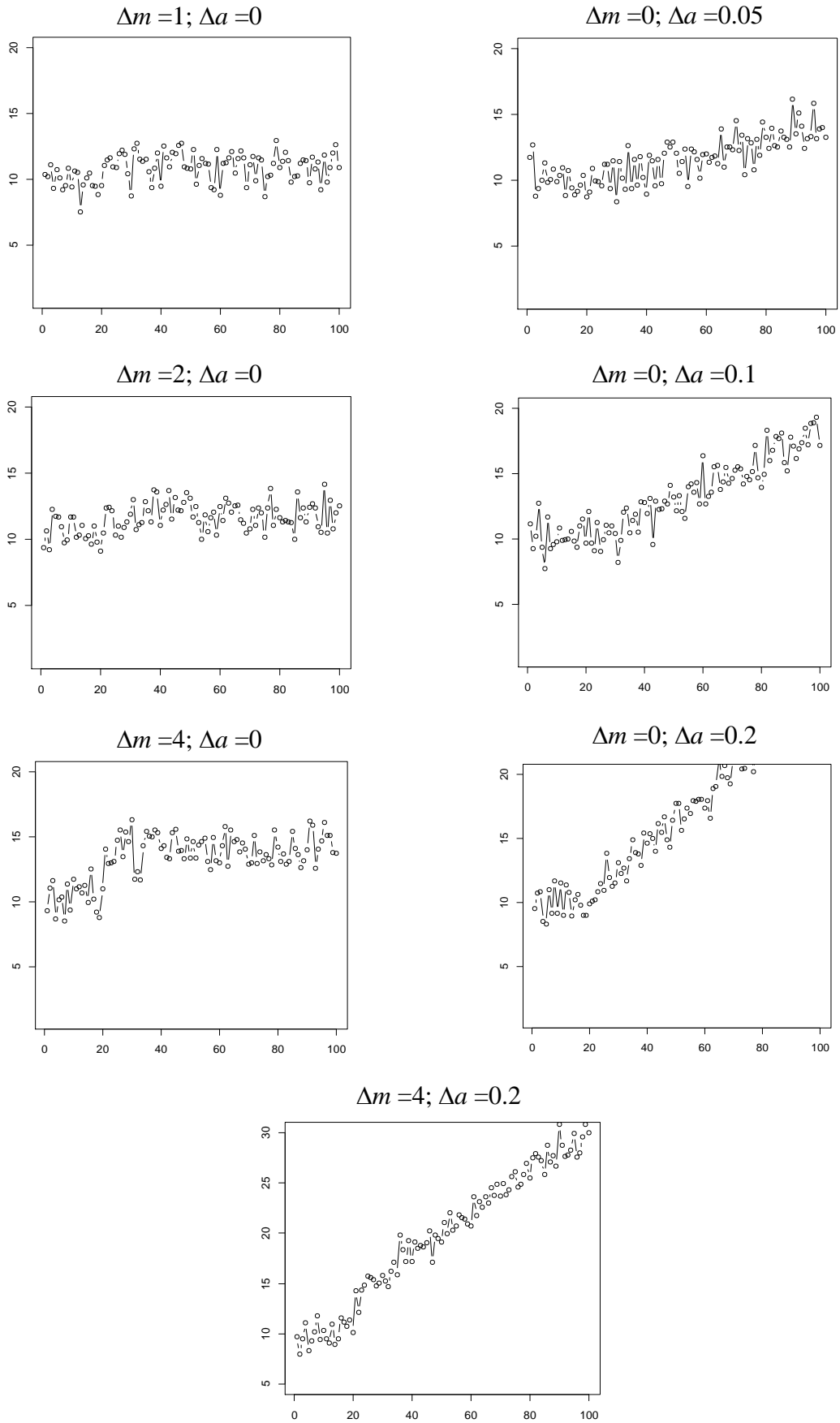
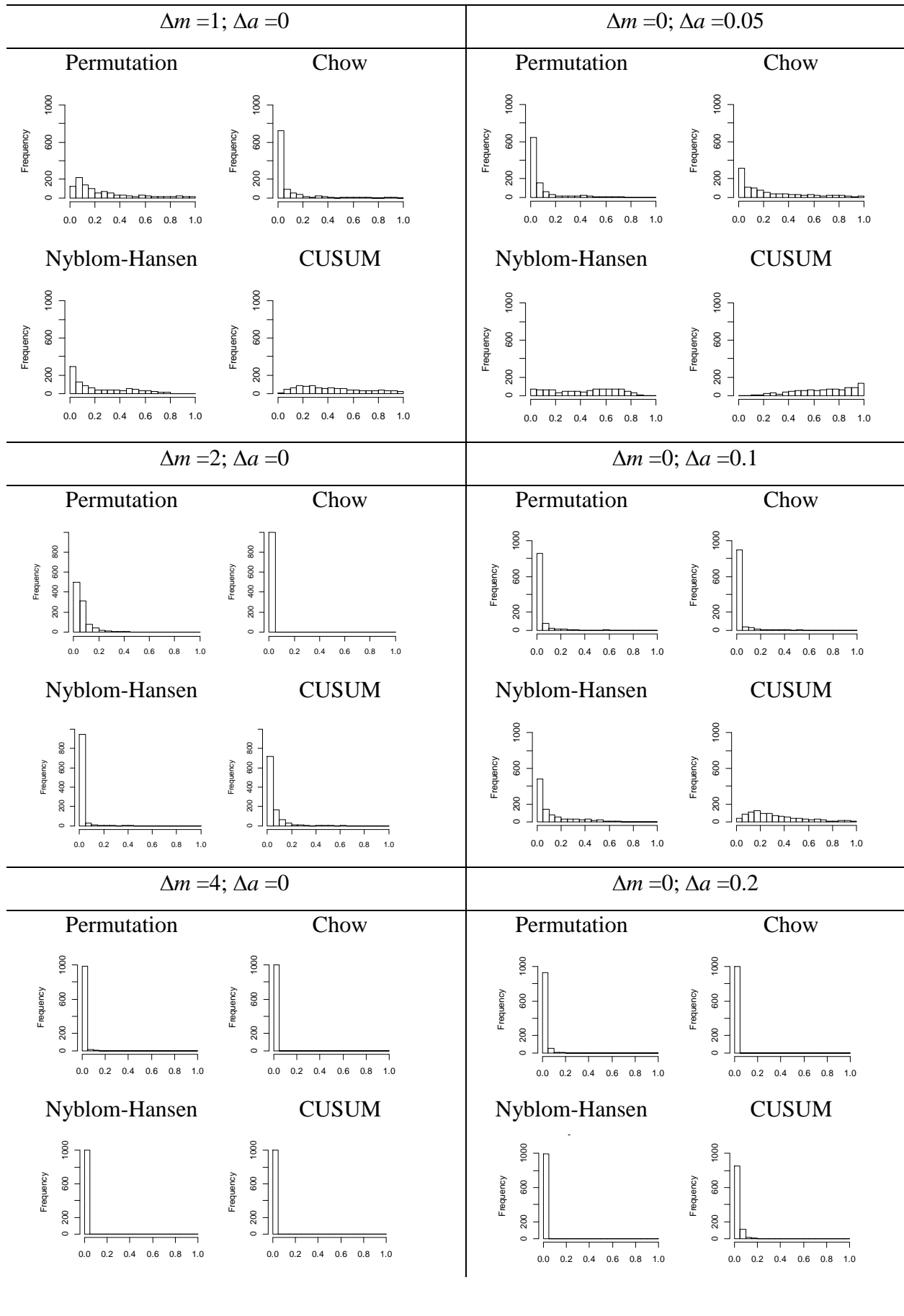


Fig. 2. Sample realization of time series.



$$\Delta m = 4; \Delta a = 0.2$$

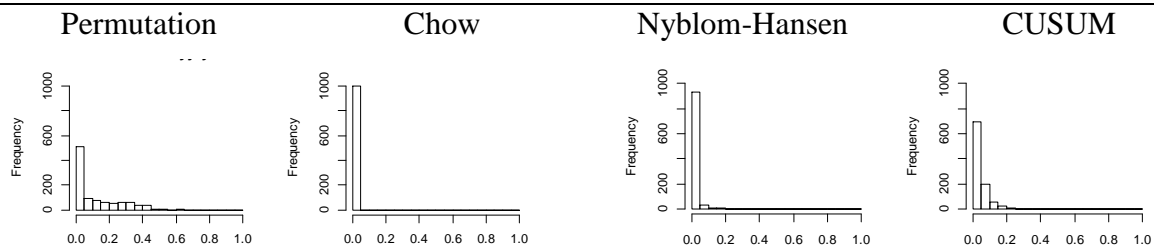


Fig. 3. Histograms of p -values.

Based on these results it can be concluded that there are situations in which the proposed method has higher power than the other tests. This can be seen when there is no change in the mean of the analyzed process but occurs the trend. The proposed method cope much better in a situation where the slope of the curve after the point of refraction is small ($\Delta a = 0.05$). The power of this assay is much greater. However, it works much worse when there is a change in the mean value. Further studies should focus on the assay design in such a way to improve its strength also in such cases.

Conclusion

The proposed procedure is used to detect structural changes in time series. Using a permutation test not meet the assumptions as when using parametric tests. In particular, the proposed method can be used to detect a trend, for example, in process control and detection of changes in the average value. Simulation studies have shown that the method is effective especially in the case of small structural changes when occurs a trend.

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