Quality evaluation of the model-based forecasts of implied volatility index

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Abstract

Influence of volatility on financial market forecasts is very high. It appears as a specific factor which causes that financial instruments' prices are highly changeable which causes problems with accuracy of their forecasts. When the market is dynamically developing, regulation of its volatility forecasts and magnification of their accuracy have particular significance, even though calculating it is complex and its values are hard to be interchangeably determined. That is the reason why model-based forecasts and especially their accuracy should be constantly modified and improved to as precisely as possible. The aim of this paper is extended analysis of the implied volatility VIX index volatility forecasts quality by examining various errors of forecasts based on the GARCH class model which allows to decide whether model-based forecasts are sufficiently accurate. These forecasts are next compared with realised VVIX index quotes from the appropriate period of time. The VVIX is established as an estimator of volatility of VIX, because financial market and model-free implied volatility appear to be the most accurate representation of this unobservable variable. It is not only the problem with volatility itself. By estimating the appropriate volatility estimator for VIX index, the result is an unobservable variable of unobservable basic instrument – implied volatility index. The chosen periods consider both calm and fluctuating periods, market tendencies (analysis of quasi-stable ups or downs) and their interdependencies.

Keywords: *quality evaluation of forecasts, GARCH modeling, implied volatility indices*

JEL Classification: C53, G17

1. Introduction

The econometric measures that allow to determine volatility level of financial instruments are very important in financial analysis. It is necessary to be aware of possible changes in trends of the instruments prices, both for long-term and immediate strategies, portfolio and single instruments.

The faster decision must be taken, the greater problem is with calculating volatility forecasts before taking the risk. Properly evaluated quality of forecasts allows to react more efficiently. Process of computing and evaluating data parameters ought to be much faster than it had been before, mainly because of rising frequency of recorded transactions and character of trading itself. This highly accelerates process of trading by shortening periods between recorded transactions to intradaily data, which are fundamentally irregularly spaced.

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This ultra-high frequency data, where spaces between recorded transactions were finally shortened to only few nanoseconds, is limited only by the number of completed transactions per given period of time (Engle, 2000). Traders are obligated to make decisions faster and while they are creating their strategies in such conditions. The adapted methods must be precise and elastic to allow changing strategies more frequently. There is also problem with volatility because of its ambiguous character as a measure. Volatility cannot be precisely and interchangeably determined as other observable variables, which causes that selecting appropriate estimator is necessary. These are only few reasons why importance of analyzing the quality of forecasts is bigger than it had been before and is still rising.

The aim of this paper is to conclude whether the conditional variances of the VIX index (commonly used abbreviation of the Volatility Index which is calculating on the Chicago Board Options Exchange) calculated using model-based forecasts are sufficiently accurate as estimator in comparison with implied volatility provided by model-free VVIX index (the abbreviation of other Volatility Index which is also calculating on the CBOE and in which the basic instrument is the VIX). The latter approach is intuitively the most appropriate estimator of volatility of VIX, because it is already the most reliable indicator of market signals. The observed interchangeable volatility values of VIX do not exist, therefore the VVIX index is assumed for the needs of this paper as observed values of VIX volatility. Evaluating the quality of model-based forecasts is given by comparison with the observed values of VVIX time series for suitable period of time. It could help to decide whether model-based approach is sufficiently correct and whether relying on it instead of on implied volatility is reasonable and adequate. The model-based forecast will be represented by GARCH class model. The quality of forecasts will be evaluated by interpreting the set of popular error types and measures outlined below in section 2.

The paper proceeds as follows. Section 2 outlines shortly the most important elements of different approaches which allow to measure volatility, both received from model-based forecasts and implied volatility. It also shows the theoretical aspects about errors useful in evaluating forecasts quality. Section 3 presents data series used for this study and some important data issues. Section 4 presents the empirical results and contains both single and complex, synthetic interpretation of these results. At the end, section 5 provides the main concluding remarks.

2. Theoretical aspects of forecasting volatility

Forecasting the main trends of future volatility level and other important tendencies and changes in prices of financial instruments, such as derivatives, or in parameters for instance as implied volatility indices, nowadays seems to be necessary or in rational approach – even obligatory. Mentioned process of receiving forecasts allows investors and analysts more or less precisely decide, whether the risk of given investment should be taken or not.

2.1 Measures of implied volatility

The instruments that measure volatility are financial instruments designed to track the value of implied volatility of some other derivative instruments, for instance, the CBOE Volatility Index (the VIX), which is computed from a weighted average of given implied volatilities of various options that are quoted on the S&P 500 Index. Implied volatility can be measured using different approaches. The model-free implied volatility presented in VIX index is for instance justifying validity of the VIX formula under the jump-diffusion process with stochastic instantaneous variance, denoted as a "SVSCJ" (Chien-Hung and Yueh-Neng, 2010).

The VVIX determines a volatility of volatility by measuring the expected volatility of price of the VIX predicted in advance with 30-days. The method of calculation in VVIX index is the same methodology as the one which is used to calculating the values of VIX index, with the main obvious difference - the basic instrument is the VIX. It is a value that is derived from the price of a portfolio which consist of liquid VIX options that are at- and out-of-the-money. This portfolio could be traded to handle the volatility risk of exposures to the VIX and to gain from risk premium which is a difference between the expected and observed volatility levels of VIX forward prices. The VVIX index could be described as a 30-day volatility, but in practice the VIX options with expiration date with 30 days forward are usually not available. The auxiliary VVIX-like values are in that case calculated from these VIX options, that their expiration dates are the two dates before and after previously mentioned 30-days period. Then the VVIX index is interpolated from those additional numbers.

The VVIX is calculated from VIX options prices using the VIX formula presented below:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \tag{1}$$

where σ is VVIX/100, T – time to expiration, F – forward index level derived from index option prices, K – strike price, K_i – strike price of ith option, ΔK_i – interval between strike prices, R – risk-free rate to expiration, $Q(K_i)$ - midpoint of bid-ask spread for each option.

It is also important that the value of VVIX index cannot be mistaken with the expected volatility of the VIX itself, because these are two different approaches use two different methods to various applications.

2.2 Model-based volatility forecasts and their quality measures

The complex and sophisticated methods used in calculating implied volatility in the VIX seems to be adequate and enough accurate to predict instrument's volatility relying only on them. In financial literature exists numerous researches that concludes that implied volatility yields superior forecasts of future volatility and in combination of approaches often implied volatility is preferred (Lamoureux and Lastrapes, 1993; Pong et al., 2004), but it is also examined, that S&P500 options market cannot anticipate any unfamiliar forward movements in variability that could not be anticipated by the model-based forecast, such as GARCH class models (Becker et al., 2007).

The main weakness of GARCH class models is that the expected variance of returns is calculated as a polynomial of its historical values – the past squared returns. On the other hand it is quite intuitive multipurpose tool to research variability. Estimated conditional variance h_t and residual of the model ε_t could be received by adjusting model to its empirical values (Piontek, 2002):

$$h_{f,t+k} = \hat{\omega} + \hat{\alpha}\,\hat{\varepsilon}_t^2 + \hat{\beta} h_t \tag{2}$$

where h_t – estimated conditional variance, ε_t – estimated model residual, $(\hat{\omega}, \alpha, \beta)$ – vector of estimated model parameters, $h_{f,\ t+k}$ – forecast of conditional variance.

In previous section, there was presented why quality of volatility forecasts is so important. The main problem with measuring volatility is that this variable is unobservable, so to evaluate the quality of forecasts a comparison with values of assumed volatility estimator is required. Because volatility in model-based approach is determined as conditional variance of returns, the natural consequence is to determine squares of returns in approach using implied volatility indices. This approach is not free of any complications (Doman and Doman, 2009).

Errors of forecasts *ex post* e_m^* is a difference between the value of the model – based forecast $\hat{y}_m^*$ – in this paper received as a result of GARCH model forecast – and observed value of the forecasted variable y_m^* (Witkowska, 2005), in this paper represented by the VVIX index, as on equation (3):

$$e_m^* = \hat{y}_m^* - y_m^* . \tag{3}$$

The set of the most important measures that have applications in evaluating quality of forecasts, both in forecasts of conditional mean and of volatility, which are also called synthetic error measures ex post outlines as follows:

- Mean squared error:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(y_{T+i} - \hat{y}_{T+i}\right)^2 , \tag{4}$$

- Median squared error:

$$MedSE = mediana\left\{\left(y_{T+i} - \hat{y}_{T+i}\right)^2\right\}_{i=1}^{N} , \tag{5}$$

- Mean error:

$$ME = \frac{1}{N}\sum_{i=1}^{N}\left(y_{T+i} - \hat{y}_{T+i}\right) , \tag{6}$$

- Mean absolute percentage error:

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{y_{T+i} - \hat{y}_{T+i}}{y_{T+i}}\right| . \tag{7}$$

These errors allow to evaluate basically the quality of forecasts. Besides these mentioned basic ones, there are many more, that are some kind of conversions and modifications of these presented above. For instance, the mean absolute error MAE – where the only difference from mentioned above ME is that the elements of subtractions are its absolute values $|y_{T+i} - \hat{y}_{T+i}|$, the root mean square error RMSE – which is a root of previously mentioned MSE and adjusted mean absolute percentage error (AMAPE) – where the only difference from the MAPE appears in its denominator where the sum consisted of $y_{T+i} + \hat{y}_{T+i}$ was used.

3. The data set

The paper shows a comparison of 3 different periods of the time series presenting VIX index forecasts and suitable VVIX index observed quotes for 3 observed periods. The GARCH models applied to calculate forecasts were fitted to 3 data sets. Each of this data sets consists of a characteristic data that represent appropriate type of data – first one was the trend of the

VIX quotes quite aggressively decreasing in period from 27th October 2008 to 4th May 2009, the second one was the trend apparently increasing in period from 15th May 2008 to 17th November 2008 and the last third period of relative static stabilization of the VIX quotes with possibly small mean value of yields variance captured in period from 18th July 2013 to 22nd January 2014.

The number of forecasts, which is equal to observed values of VVIX in each of these 3 periods was 55, which gives about 11 weeks (equal to less than 3 months). Only the working days were considered. The time series both of the VIX and the VVIX indices used for the calculations were downloaded from the official website of CBOE, where the time series of appropriate quotes are available and daily uploaded.

4. Analysis and interpretation

The main macroeconomic reason for decreasing trend in the end of 2008 to May 2009 could be that financial markets started to receive stabilization after the shocks after crisis (the more rumors and negative information such as crisis potential consequences, the higher level of volatility on the financial markets). This could describes why the increasing tendency had place in the early 2008, where the crisis was causing indirectly still the bigger and bigger volatility level. Nowadays, the volatility is much lower, because of the lack of recent accumulation of negative information, so in given comparison period from 2013 to 2014 appears as the most stable.

As it was declared in introduction (section 1), the aim of this paper is to evaluate quality of forecasts by comparing the results of VIX volatility calculated using conditional variances in GARCH class model with the implied volatility index observed for the same period by analyzing the VVIX index.

To calculate the VIX index model-based volatility, after pursuing the appropriate comparison, the GARCH(p,q) models that have been chosen was in all three cases the GARCH(1,1) for the most stable period, the increasing period and the decreasing period. This were a GARCH models with the Gaussian distribution. To select the best fitted model, models GARCH(1,1), GARCH(1,2), GARCH(2,2), GARCH(2,3) and GARCH(3,3) were compared with each other for each period. The main reason why model was chosen was the comparison of its p-value (the less p-value the better), standard error (the less standard error the better) and t-Statistic. The received forecasts of VIX were compared with the VVIX in suitable periods to calculate the ex post forecast errors, which are presented in Table 1.

Forecast error	Increasing period	Decreasing period	Stable period
MSE	0.0084	0.0079	0.0026
MedE	0.0028	0.0001	0.0052
ME	0.0088	0.0002	0.0021
RME	0.0094	0.0046	0.0461
MAPE	685.91%	117.68%	17.94%
RMSE	243.55%	17.56%	8.30%
AMAPE	65.93%	59.58%	49.30%
MAE	0.0213	0.0209	0.0009

Table 1. Calculated errors of model-based forecasts.

The results of evaluating quality of forecasts proved that model-based forecasts have no possibilities to equal to implied volatility models. The conditional variances are mainly based on historical dataset and do not reflect the other effects, because they are not designed in connection with the other parameters, for instance expiration date of instrument or its market price (on the other hand implied volatility used in the VVIX index does). It was also quite simple to predict that divergence between the calculated values and observed ones will be high because both represent different methodology of calculating unobserved parameter – the volatility. The model-based forecasts are not recommended to be applied in high-frequency datasets and other instruments and strategies that demand taking the fast and efficient short-term decisions, always when the better and more efficient equivalents are constantly available on market. Implied volatility is usually better from most of model-based forecasts, such as GARCH class models, as it was previously mentioned in literature (Lamoureux and Lastrapes, 1993; Pong et al., 2004) in section 2.

Despite the fact that recommendation of model-based forecasts in predicting the future tendencies of volatility of volatility is quite negative, profound analysis of calculated errors both separately and considering the relations between them, could be efficient and valuable source of information about the reasons of differences and tendencies in compared datasets. Information received from these forecast errors are not multiplying the same information. The modifications of forecast errors definitions essentially change their meaning. Each of them has its own different interpretation but the most reliable is analyzing these parameters together including their mutual dependencies. For instance, the MSE, MAE and RMSE errors give quite similar information about the efficiency of forecasts, but each of them react in different way for the untypical observations. The values of these 3 errors that are close to each other, which indicates that in forecasts series do not appear very untypical and extreme observations. If MSE is smaller than MAE, as it takes place in decreasing and stable periods, it indicates that the single big-value errors appeared in these series. The values were quite similar, but the MSE was much smaller than MAE, so the extreme and untypical values have appeared, but they were in great minority. There are also much smaller ME than MAE values, what indicates that the underestimated and the overestimated values were quite equal in two periods - decreasing and stable ones, but in the increasing period, where both of errors are relative bigger to their absolute values, which seems that forecasts probably are systematically underestimated (because the signs of these errors are positive).

Conclusion

The implied volatility has more information impact with higher divergence of each parameters than model-based forecasts of volatility. It does not matter whether the model is GARCH class or Stochastic Volatility, because the volatility that depends on more than historical values and reflects the other important parameters of basic instruments is a big advantage. There were also found that newly CBOE markets are informational efficient and both point and interval forecasts of implied volatility are statistically significant (Konstandinidi et al., 2008). Model-free implied volatility captured in the VVIX index reflects better the financial market factors as volatility of volatility estimator than model-based ones. The other reason for existing disparity, is that because the forecasts measure the volatility of volatility and the VVIX index quotes are treated as observed values of realized volatility of the VIX index, because the volatility is unobservable variable and it could not be interchangeably determined, but the assumption about appropriate estimator is required.

The selected type of error is important in global interpretation of the received forecasts and each of these measures allows to receive quite different information about the forecasts as a series. It is also good source of information about mutual dependencies that appear between them. Also the information could be achieved from the sign of the error, its value (whether it is big or small) and even from its relation with the other parameters (whether it is bigger or smaller than rest of the measures). The similarity of errors is purposeful to let to focus on the single differences that may reflect on behavior of all dataset and its reactions for the single factors, but simultaneously these outwardly similar measures do not only duplicate the same information as it could seem to appear.

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