The use of Markov chains in the social convergence analysis

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Abstract

The main goal of this article is to analyze the social convergence process using Markov chains. In this research, term social convergence refers to the reduction of dispersion in the standard of living among countries. The use of Markov chains in the social convergence analysis allowed describing movements of observational units within the distribution and providing more details about the mechanism of the convergence process. A taxonomy spatial measure of development, proposed by Antczak, was used as the standard of living approximation. The use of a new approach allowed to explain the disparities in the analyzed phenomenon by taking into account the immeasurable spatial factor. The analysis included 24 Member States of the European Union over 1995-2012 period. The results of the analysis indicate the existence of convergence in the European Union.

Keywords: *Markov chains, social convergence, taxonomy spatial measure of development, standard of living*

JEL Classification: C43, I31

1. Introduction

This research is a continuation of the author's previous researches on social convergence in the European Union (Kuc, 2014a; Kuc, 2014b). Similarly as in the prior articles the social convergence is understood as the decreasing of spatial disproportion in the standard of living among countries. Compared to author's previous studies, two modifications were made. Firstly, in this paper, the convergence is analyzed using the transition matrix of the Markov chains. Secondly, to quantify the standard of living, the idea of taxonomy spatial measure of development proposed by Antczak (2013) was used.

To test for the existence of social convergence the transition matrix of Markov chains was implemented. This tool was originally used in the analysis of income convergence conducted by Fingleton (1997), Rey (2001), Le Gallo (2004), Monfort (2008), Wójcik (2008), Bosker (2009), Łaźniewska and Górecki (2012), Decewicz (2013). However, it seems that it can be as well successfully adapted to the analysis of social convergence.

The spatial factor was included in the standard of living calculation. Since the use of a regional dataset implies consideration of the possibility that observations may not be independent, as a result of the inter-connections between neighbouring regions (Buccellato, 2007). Moreover, as Waldo Tobler said "everything is related to everything else, but near things are more related than distant things" (Tobler, 1970). Finally, empirical analyses that

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have ignored the influence of spatial location may have produced biased results (Fingleton and Lopez-Bazo, 2006).

The study was conducted for 24 European Union member countries (Cyprus, Malta and Luxemburg were excluded due to the lack of data) in the years 1995-2012. Empirical material was taken from GMID Passport Euromonitor database.

2. Markov chains in the convergence analysis

In addition to classical methods of analyzing the beta, sigma or gamma-convergence (Kusideł, 2013) Markov chains can be also used to empirical verification of this phenomenon. The analysis of the convergence process with the use of Markov chains should be started from dividing the observations into a set of *m* non-overlapping classes:

$$\bigcup_{i=1}^{m} K_i = K \qquad i, j = 1,2,...,m. \tag{1}$$

In the next step of the analysis the transition probability is calculated, i.e. the probability that given object will move from group *i* to group *j*:

$$\pi_{ij} = P\left(z_{r,t} \in K_j \middle| z_{r,t-1} \in K_i\right) \qquad i, j = 1,2,...,m \tag{2}$$

where π_{ij} – conditional probability of transition of the object *r* from the group *i* to the group *j*.

It is assumed that the Markov chain is an ergodic chain. Thus, it is possible for each object to shift to any group (both lower and higher) in a finite number of transitions (Podgórska et al., 2000). The estimated values of π_{ij} form the transition matrix Π:

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1m} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{m1} & \pi_{m2} & \cdots & \pi_{mm} \end{bmatrix}. \tag{3}$$

The following equation can describe the evolution of the distribution over time:

$$p = \Pi'^{T} p \tag{4}$$

where p – steady state vector, Π – transition matrix.

If Π is the transition probability matrix of an ergodic Markov chain, then the chain is characterised by a stationary distribution corresponding to a steady-state towards which the distribution will converge in time (Monfort, 2008). On the basis of the transition probability matrix Π the half-life indicator and the speed of convergence can be calculated:

$$half-life=\frac{-\ln 2}{\ln|\lambda_2|} \tag{5}$$

where λ_2 – the second eigenvalue of matrix Π.

The high value of the half-life indicates a rapid convergence to steady-state. Pellegrini (2002) developed a stability index S, which determines the probability that the object will remain in the same class:

$$S=\frac{tr(\Pi)}{n} \tag{6}$$

where tr(Π) – the trace of matrix Π, n – the dimension of matrix Π.

The high value of S indicates that the process is stable, so the probability to move from one group to another is small.

3. Taxonomy spatial measure of development as the standard of living approximation

In this research standard of living refers to the level of wealth, comfort, material goods and necessities available to a certain socioeconomic class in a certain geographic area (Bywalec and Wydymus, 1992). The taxonomy spatial measure of development is used as the standard of living approximation. Using the idea proposed by Antczak (2013) the standard of living was calculated as follows:

1. Setting the wide set of diagnostic variables that are crucial to describe the analyzed phenomenon (112 variables);
2. Removing variables that do not meet the formal correctness conditions (Zeliaś, 2004);
3. Testing the existence of spatial autocorrelation using Moran's I statistic:

$$I=\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \tag{7}$$

where w_{ij} – the weight between the country *i* and *j*, x_i – the value of a variable at a particular location.

In Table 1 variables for which Moran I was statistically significant in 2012 are written in italics.

Domain	**Variables**
Population	*total fertility rate*; old age dependency ratio
Labour market and job security	*unemployment rate*; the employment rate; number of deaths due to the accident at work per 100 000 inhabitants
Health and social care	*number of doctors per 100 000 inhabitants*; number of nurses per 100 000 inhabitants; number of hospital beds per 100 000 inhabitants; *number of deaths due to tuberculosis per 100 000 inhabitants*; number of deaths due to diabetes per 100 000 inhabitants; infant mortality rate; *number of new AIDS cases per 100 000 inhabitants*; obesity rate
Education	*number of university students per 1000 inhabitants*; number of academic teachers per 1 student
Leisure time	annual cinema trips per capita, *number of hotels per 1000 inhabitants*
Living conditions	The number of newly build dwellings per 1000 households
Transport and communication	*number of newly registered passenger cars per 1000 inhabitants;* airline passenger transport in passenger-km per capita*; railway transport in passenger-km per capita; density of road network; proportion of paved roads in total road network*; number of mobile phone subscribers per 1000 inhabitants, *percentage of population with access to the Internet*
Social security	*corruption perception index; number of murders per 100 000 inhabitants*; number of drugs related crimes per 100 000 inhabitants; *number of suicides per 100 000 inhabitants,* number of divorces per 1000 inhabitants
Population incomes and expenditures	*total savings as a percentage of disposable income; tax and social contributions as a percentage of gross income; the inflation rate;* wage per hour in manufacturing (in euro - fixed exchange rate 2012)
Natural environment	particulate matters emission in micrograms per square meter; *nationally protected areas as percentage of total land*; carbon dioxide emission in kg per capita; *forest land as percentage of total land*; water pollutant emission in kg per 1000 inhabitants

Table 1. The set of diagnostic variables.

4. Calculating the taxonomy spatial measure of development for every domain of the standard of living according to the formula:

$$z_{iqt}^{sp} = \frac{\sum x_{ijt}^{'} + \sum x_{ijt}^{'*}}{k} \quad q = 1,...,10 \tag{8}$$

where z_{iqt}^{sp} – the synthetic variable for the country *i* calculated on the basis of variables belonging to *q* group in year *t*, k – the number of variables in given group, $x_{ijt}^{'}$ – the value of normalized variable without spatial character, calculated as:

$$x_{ijt}^{'} = \frac{x_{ijt} - \min x_{ijt}}{\max x_{ijt} - \min x_{ijt}}, \tag{9}$$

x_{ijt} – the value of variable *j* in the country *i* in year *t*,

$\min x_{ijt}$ – the minimum value of variable *j* in year *t*,

$\max x_{ijt}$ – the maximum value of variable *j* in year *t*,

$x_{ijt}^{'*}$ – the value of normalized variable with spatial character, calculated as:

$$x_{ijt}^{'*} = \frac{x_{ijt}^{*} - \min x_{ijt}^{*}}{\max x_{ijt}^{*} - \min x_{ijt}^{*}}, \tag{10}$$

$x_{ijt}^{*} = W x_{ijt}$

W – the spatial weight matrix:

$$W = \lfloor w_{ij} \rfloor \tag{11}$$

$$w_{ij} = \begin{cases} 1, when\ \ i\ \ and\ \ j\ \ have\ \ common\ \ border. \\ 0, otherwise \end{cases}$$

5. Calculating the standard of living measure as the average of synthetic variables for each domain:

$$z_i^{sp} = \frac{1}{p}\sum_{q=1}^{p} z_{iqt}^{sp} \qquad i = 1, 2, ..., n \qquad q = 1, 2, ..., p. \tag{12}$$

4. Empirical analysis

Estimated values of z_i^{sp} were basis to divide countries into four categories. The results are included in Table 2, in brackets are given values of the standard of living synthetic measure (SoL).

Group of countries	1995	2012
The highest standard of living $z_i^{sp} \geq \bar{z}^{sp} + s_{z^{sp}}$	IE (0.5403), NL (0.5315), UK (0.5221), DE (0.4755)	FI (0.5197), SE (0.4977), UK (0.4875), IE (0.4857)
Medium standard of living $\bar{z}^{sp} + s_{z^{sp}} > z_i^{sp} \geq \bar{z}^{sp}$	SE (0.4587), FI (0.4490), BE (0.4383), PT (0.4349), AT (0.4200), FR (.04035), DK (0.3978), ES (0.3783),	AT (0.4690), NL (0.4559), DK (0.4529), DE (0.4496), SK (0.4248), IT (0.4124), FR (0.4099), BE (0.4081), ES (0.4050)
Low standard of living $\bar{z}^{sp} > z_i^{sp} \geq \bar{z}^{sp} - s_{z^{sp}}$	CZ (0.3569), IT (0.3543), GR (0.3329), SI (0.3293), SK (0.3242), PL (0.3025), LT (0.2822), HU (0.2811), EE (0.2751)	CZ (0.3799), PL (0.3550), PT (0.3462), SI (0.3183), GR (0.3009), RO (0.2984)
The lowest standard of living $z_i^{sp} < \bar{z}^{sp} - s_{z^{sp}}$	RO (0.2232), BG (0.2130), LV (0.2098)	EE (0.2935), LT (0.2820), LV (0.2751), HU (0.2725), BG (0.2338)

Table 2. Group of countries due to the standard of living.

Based on the results presented in Table 2 the transition probability matrix and steady state vector were calculated. The results of the analysis are included in Table 3.

The diagonal values at the transition probability matrix (Table 3) are high, that suggest that there is a high probability remaining in the same group as in the first year of analysis. That situation is also confirmed by the high value of S index that means that the probability of remaining in the same class is 55.9%. Analyzing the value of transition probability matrix one the 66,6% of countries that had the lowest standard of living remain in the same group in 2012. Moreover, up to 33% of countries that were originally in the group with low standard of living have moved in 2012 to the group of countries with the lowest standard of living. The probability of remaining in the group of countries with low standard of living is 44.4%, which is also quite high. Only 22% of countries were able to move from low to medium standard of living. A slightly more optimistic is situation in countries that in year 1995 were included in the medium standard of living group, 62.5% of them remain in the same group and 25% of them moved to the highest standard of living group. That means that only 12.5% of countries with initially medium standard of living have joined the group with low standard of living. Interesting situation can be observed in the group of countries with the highest standard of living; only half of them remain in the same group while the other half dropped to the group of medium standard of living.

			2005			
1995	SoL	Percentage of countries	The lowest	Low	Medium	The highest
	The lowest	12.5%	0.666	0.333	0.000	0.000
	Low	37.5%	0.333	0.444	0.222	0.000
	Medium	33.3%	0.000	0.125	0.625	0.250
	The highest	16.7%	0.000	0.000	0.500	0.500
		Summary statistics				
			The lowest	Low	Medium	The highest
Stationary distribution			0.19	0.38	0.21	0.21
Half-life			5.15			
S			0.559			

Table 3. Transition probability matrix.

The process of catching up may be observed, and the stationary distribution indicates it. The distribution is likely to feature smaller disparities in long-run than the initial one with a concentration of countries in the group ‘medium standard of living’. However, the peace of convergence is very slow with a half-life of higher than 5 for 18 years period.

Conclusion

In this research, the existence of social convergence was tested with the use of transition matrix from Markov chains. The Markov chains are relatively good tools in club convergence analysis. Club convergence is the situation in which a set of countries may converge with each other (tend to have a common steady state) but they do not converge across different sets of countries. So far, relatively small attention has been paid to identify the existence of social convergence clubs among European Union. The research indicated the convergence toward stationary distribution; however the peace of convergence is very slow. The examination of the distribution showed changes, which were not detected in the classical approach. Therefore, it should be aware that the Markov chains allow to describe a slightly different process than the traditional methods of testing convergence. For that reason, it is hard to point out which of these methods is more effective. Farther, all those methods should be rather used complementarily to provide complete and in-depth analysis.

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