

## Factor shares and income and wealth inequalities

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### Abstract

In the paper, we summarize the recent findings on the worldwide evolution of the capital and labour shares of output in the last few decades. We also study the consequences of the observed decline in the labour share for income and wealth inequalities. We employ a standard DSGE model with heterogeneous agents and Cobb-Douglas production technology that matches the observed wealth distribution in the United States. Then, we change the values of the parameter that governs the capital share in the model and study its impact on the inequalities measured by the Gini coefficients and the wealth/income shares of the top 1%. The results show that the increase of one percentage point in the capital share raises the Gini coefficient of the income distribution of about 0.008, on average. At the same time, the changes in the factor shares remain almost neutral for the analogous wealth inequality measures.

**Keywords:** labour share, inequality, heterogeneous agent model, production function

**JEL Classification:** E25, C63

### 1. Introduction

At least since Kaldor (1957), it was common to assume that the factor shares of output are approximately constant. However, recent studies document the significant decline in the labour share that started around 1980's (Rodriguez and Jayadev, 2010; Karabarbounis and Neiman, 2014; Piketty, 2014). The drop is observed in many economies, both developed and developing. For example, in the United States, the labour share had been quite stable before 1980s and fluctuated around 64%. Then, it started to decrease by almost two percentage points per a decade, on average. In Poland, the phenomenon is even more striking. The labour compensation dropped from 70% in 1992 to 52% in 2013 – a decline of almost ten percentage point per a decade.

The observations pose a real challenge for economists to look for the causes and the consequences of the labour share decline. The decrease of relative prices of investment goods (Karabarbounis and Neiman, 2014) or globalization (Guscina, 2007), among others, are proposed as the possible explanations for the phenomenon. On the other hand, the literature points out that the changes in the labour share of output are likely to affect primarily the two central economic phenomena – growth and inequalities. Nonetheless, neither the causes nor the consequences have not been yet examined in the literature thoroughly.

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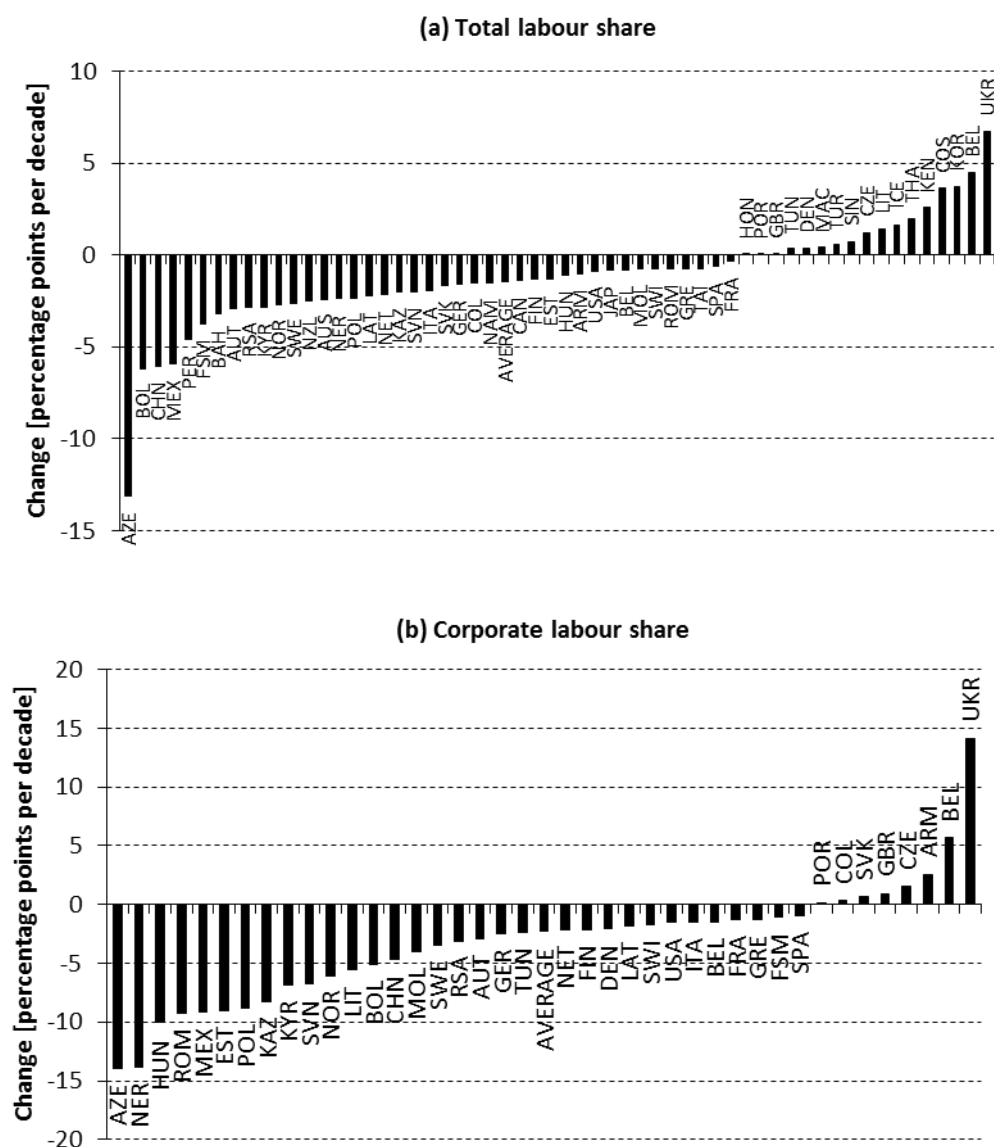
In the paper, we focus on the consequences and study, within a very simplified framework, the impact of the labour share decline on the income distribution. We also try to assess to what extent the increase in the income inequalities resulted from the labour share decrease translates into the rise of the wealth concentration. We use a simple Bewley-style (see Bewley, 1993; Aiyagari, 1994) heterogeneous agent model, where *ex ante* identical agents face an idiosyncratic labour market risk. Because of different employment histories, *ex post*, they hold different amounts of wealth. Following Krusell and Smith (1998), we assume that agents can differ with regard to the discount coefficient which allows the model to roughly match the wealth inequalities observed in the United States. The production sector utilizes the constant returns-to-scale Cobb-Douglas production technology with the constant parameter governing the capital share of output. In the paper, we basically investigate the distributional consequences of changing its values.

Our results can be summarized as follows. We find that the increase of one percentage point in the capital share raises the Gini coefficient of the income distribution by 0.008, on average. However, the Gini coefficient for wealth remains virtually unchanged. Similar conclusion emerges when we study the income and wealth shares of the top 1% agents.

The rest of the paper is organized as follows. Section 2 presents the evidence of the labour share decrease in the last decades. In section 3, we briefly introduce the model. Finally, section 4 contains the results of the simulation studies.

## **2. Evolution of the labour share**

Figure 1 shows the evolution of the labour share across countries over the last few decades. Using the data collected by Karabarbounis and Neiman (2014), we calculated the average change in the labour share per a decade for the countries where at least 15 annual observations were available. The longest series start at 1975 and contain almost 40 observations. The total labour shares presented in panel (a) are calculated for the whole economies, whereas the corporate labour shares (panel (b)) account for the corporate sector only. The former measure is more representative, but it suffers from the problems with classification of the incomes of sole proprietors and unincorporated businesses. In these cases, it is hard to distinguish between incomes earned by capital and labour and therefore they are artificially imputed by a statistical procedure. This is not the case for the corporate sector, where the capital and labour compensations are well defined.



Note: The values are taken from Karabarbounis and Neiman's (2014) dataset. The first three letters are used to abbreviate the country's names, or, if it would be ambiguous, 3-letter UN codes.

**Fig. 1.** Trends in the total (a) and corporate labour share (b).

Both measures, however, clearly document that the labour share decreased in most countries. More precisely, we observe the decline of the total labour share in 40 out of 57 reported countries. On average, the labour compensation decreased by one and a half percentage point per a decade. Azerbaijan, Bolivia, China and Mexico are the countries with the rapidest drop – more than five percentage points per a decade. On the other extreme, Ukraine and Belgium exhibit the average increase of the labour share of about five percentage point.

The corporate labour share declined in 33 out of 41 countries and by more than two percentage points, on average. Exactly as for the previous measure, Azerbaijan and Ukraine exhibit the sharpest decline and rise, respectively. The corporate sector data also reveals a significant drop in the labour share in Poland, which reaches almost ten percentage points per a decade. Similar decline is observed in some other Central and Eastern European countries like Hungary, Romania and Estonia.

### 3. Model

In the paper, we use a standard, stochastic, heterogeneous agent model introduced by Bewley (1993) and Aiyagari (1994). Following Krusell and Smith (1998), we extend it to the heterogeneous discount rates which allows to roughly match the wealth inequalities observed in the United States.

#### 3.1 Consumer's decision problem

There is a continuum of infinitely-lived heterogeneous consumers, who differ in terms of wealth, the labour market status and the discount coefficient. The employed agents receive the wage  $w$  that is equal to the marginal productivity of labour. The unemployed consumers engage in a home production, which provides the income being a small fraction  $\theta$  of the wage. Additionally, the consumers receive the interest  $r$  on their capital holdings lowered by the capital depreciation rate  $\delta$ . The consumer's decision problem can be written as follows:

$$\max_{c_t, k_{t+1}} \left\{ E_t \sum_{h=0}^{\infty} \left( \prod_{i=1}^h \beta_{t+i-1} \right) \ln(c_{t+h}) \right\}, \text{ s.t. } k_{t+h+1} = (1 - \delta + r_{t+h})k_{t+h} + \psi_{t+h}w_{t+h} - c_{t+h}, k_{t+h+1} \geq k_{min} \quad (1)$$

where  $c$  stands for the consumption,  $k$  for capital,  $\beta$  for the discount coefficient,  $E$  for the expectation's operator, and  $\psi$  represents the labour-related income:

$$\psi_t = \begin{cases} 1 & \text{if an agent is employed} \\ \theta & \text{if an agent is unemployed} \end{cases} \quad (2)$$

The parameter  $k_{min} \leq 0$  sets the debt limit.

The problem can also be stated in the simpler form using the Bellman's optimality principle, where, for clarity, we omit the time subscripts and denote the next period variables with primes:

$$V(k, \psi, \beta) = \max_{k'} \left\{ \ln[(1 - \delta + r)k + \psi w - k'] + \beta E[V(k', \psi', \beta') | k, \psi, \beta] \right\}. \quad (3)$$

$V$  represents the value function.

### 3.2 Production sector

The production technology is described by a standard, constant return-to-scale Cobb-Douglas production function:

$$Y = K^\alpha L^{1-\alpha} \quad (4)$$

where  $K$  and  $L$  denote the aggregate capital and labour, respectively. In other words,  $K$  equals the average consumer's wealth and  $L = 1 - u$ , where  $u$  is the unemployment rate in the model. The parameter  $\alpha$  represents the capital share of output. The factor's prices equal their marginal productivities:

$$r = \alpha K^{\alpha-1} L^{1-\alpha}, \quad (5)$$

$$w = (1 - \alpha) K^\alpha L^{-\alpha}. \quad (6)$$

### 3.3 Stochastic shocks

There are two stochastic shocks in the model: the labour market status  $\psi$  and the discount coefficient  $\beta$ . Dynamics of both shocks is described by two independent homogenous Markov chains. For the labour market status, there are two states with the transition matrix:

$$P_\psi = \begin{bmatrix} p_{ee} & p_{eu} \\ p_{ue} & p_{uu} \end{bmatrix}$$

where  $p_{ij}$  denotes the probability of moving from state  $i$  to  $j$ . For the discount coefficient, we assume three states with the analogous transition matrix  $P_\beta$ .

### 3.4 Calibration of the parameters

To calibrate the model's parameters, we follow Krusell and Smith (1998). Basically, we want to match the wealth distribution observed in the United States, because, unfortunately, there are no reliable data on the wealth distribution in Poland. A period in the model corresponds to one quarter. In the baseline calibration, we use the standard capital share of output parameter  $\alpha = 0.36$ . The depreciation rate is set at  $\delta = 0.025$ , and the replacement rate for the unemployed agents equals  $\theta = 0.09$ .

The entries of the Markov chain transition matrix for the labour market status are calibrated to match the average unemployment duration of two quarters and the average unemployment rate  $u = 7\%$ . As a result, we have:

$$P_\psi = \begin{bmatrix} 0.95 & 0.05 \\ 0.5 & 0.5 \end{bmatrix}.$$

For the discount coefficient  $\beta$ , we use three values: 0.9858, 0.9894 and 0.993. The transition probabilities are set to ensure that (i) 80% of agents is characterized by the middle value and 10% by the extreme ones, (ii) direct transitions between the extreme states do not occur and (iii) the average duration of the states is 50 years. Therefore, the transition matrix looks as follows:

$$P_{\beta} = \begin{bmatrix} 0.9996 & 0.0004 & 0 \\ 0.0006 & 0.9988 & 0.0006 \\ 0 & 0.0004 & 0.9996 \end{bmatrix}.$$

It should also be added that in the baseline calibration the debt limit is set at  $k_{min} = -5.35$ , which means that the debt cannot exceed the quarterly wage more than 2.25 times. As a result, about 11% of the population in the model holds negative wealth, which is close to the fraction for the United States.

The numerical algorithm used to calculate the stationary distribution of wealth in the model is not straightforward, and, due to limited space, is not discussed here. The procedure employs the Euler equation iteration algorithm developed by Maliar et al. (2010). More details can be found in Acedański (2015).

#### 4. Results

Basically, to assess to what extent the labour share decline affects the income and wealth distributions, we calculate the stationary distributions for the model described above with different values of the parameter  $\alpha$ . Besides the baseline value  $\alpha = 0.36$ , we also consider values from the set  $\{0.31; 0.41; 0.46; 0.51\}$ . Because even small alternations of the parameter result in substantial changes in the aggregate wealth, which translates into variations of the wage and, to much smaller extent, the interest rate, one has to adjust the debt limit  $k_{min}$ . Therefore, we always alter  $k_{min}$  to keep its relation to the wage constant.

The results of the simulation study are presented in Table 1. We report the Gini coefficients for the income and wealth distributions, the fraction of agents with negative wealth holdings as well as the wealth share of top 1%, 5%, 10%, 20% and 30% of agents. Analogously, we also include the income share of the top 1% of agents. Finally, we show the empirical counterparts of the characteristics taken from Krusell and Smith (1998).

First, we can assess the quality of matching the wealth and income distributions for the baseline calibration. The model generates slightly higher wealth inequalities than observed. More precisely, the Gini coefficient for the wealth in the model equals 0.845, whereas in the

data it is 0.79. At the same time, the income inequalities in the model are too low (the Gini coefficient equals 0.329) compared to the observed ones (0.47). If we look at the wealth concentration among the wealthiest agents, one can notice that it is slightly understated for the top 1% (26.5% of the total wealth in the model and 30% in the data) and overstated for the other groups, especially for the top 10% (76.4% in the model and 64% in the data).

$\alpha$	Gini (wealth)	Gini (income)	Negative wealth [%]	Top 1% (wealth)	Top 1% (income)	Top 5% (wealth)	Top 10% (wealth)	Top 20% (wealth)	Top 30% (wealth)
<b>Data</b>	<b>0.790</b>	<b>0.470</b>	<b>11.0</b>	<b>30.0</b>		<b>51.0</b>	<b>64.0</b>	<b>79.0</b>	<b>88.0</b>
0.31	0.846	0.291	12.6	26.2	8.8	58.8	77.0	90.6	93.8
<b>0.36</b>	<b>0.845</b>	<b>0.329</b>	<b>11.3</b>	<b>26.5</b>	<b>10.1</b>	<b>58.5</b>	<b>76.4</b>	<b>90.7</b>	<b>94.1</b>
0.41	0.843	0.368	10.2	26.5	11.4	58.0	75.7	90.6	94.5
0.46	0.842	0.406	9.2	26.7	12.8	57.7	75.1	90.4	94.8
0.51	0.836	0.442	8.2	25.0	13.2	56.1	73.6	89.6	94.9

**Table 1.** Results for the baseline calibration.

The main results show that the income inequalities rise considerably as the capital share grows. The Gini coefficient increases from 0.291 for  $\alpha = 0.31$  to 0.442 for  $\alpha = 0.51$ . In other words, the decline in the labour share of one percentage point translates into the increase of the Gini coefficient by 0.008, on average. A similar trend exists for the top income shares. For example, the income share of the top 1% rises from 8.8% to 13.2%, so by 0.2 percentage point for each one percentage point drop of the labour share.

Surprisingly, despite the sharp increase in the income inequalities, the wealth inequalities remain virtually unchanged. The Gini coefficient of the wealth distribution fluctuates between 0.836 and 0.846, but reaches the minimum for  $\alpha = 0.51$ . Initially, the wealth share for the top 1% rises from 26.2% for  $\alpha = 0.31$  to 26.7% for  $\alpha = 0.46$ , but for  $\alpha = 0.51$  it drops to 25%.

In the next step, we drop the assumption about the logarithmic utility. Instead, we consider the CRRA utility function of the form:

$$u(c) = \begin{cases} \frac{c^{1-\gamma} - 1}{1-\gamma} & \text{if } \gamma > 1 \\ \ln(c) & \text{if } \gamma = 1 \end{cases} \quad (7)$$

where  $\gamma$  stands for the relative risk aversion coefficient. The results for  $\gamma = 2.5$  are shown in Table 2. The general conclusions do not change, however. The Gini coefficient of income

soars from 0.257 to 0.385 and the income share of the top 1% jumps from 6.2% to 9.7%. On the other hand, the Gini coefficient for wealth drops slightly, from 0.746 to 0.731, and the wealth share of the top 1% fluctuates between 17.7% for  $\alpha = 0.31$  and 19% for  $\alpha = 0.46$ .

$\alpha$	Gini (wealth)	Gini (income)	Negative wealth [%]	Top 1% (wealth)	Top 1% (income)	Top 5% (wealth)	Top 10% (wealth)	Top 20% (wealth)	Top 30% (wealth)
<b>Data</b>	<b>0.790</b>	<b>0.470</b>	<b>11.0</b>	<b>30.0</b>		<b>51.0</b>	<b>64.0</b>	<b>79.0</b>	<b>88.0</b>
0.31	0.746	0.257	13.1	17.7	6.2	42.4	59.3	77.7	86.6
<b>0.36</b>	<b>0.738</b>	<b>0.288</b>	<b>11.1</b>	<b>17.9</b>	<b>7.1</b>	<b>42.1</b>	<b>58.6</b>	<b>77.1</b>	<b>86.4</b>
0.41	0.735	0.320	9.8	18.2	8.0	42.2	58.4	76.7	86.3
0.46	0.736	0.354	8.9	19.0	9.2	42.9	58.7	76.6	86.4
0.51	0.731	0.385	7.9	18.0	9.7	42.3	58.0	76.0	86.0

**Table 2.** Results for the calibration with higher risk aversion.

Finally, we also study the model where agents cannot incur debts, which corresponds to setting  $k_{min} = 0$ . The results are presented in Table 3. Now, both, the income and wealth inequalities raise similarly. The observation supports the thesis that the behaviour of agents with the negative wealth holdings is a key factor behind the insensitivity of the wealth inequalities to the changes in the factor shares.

$\alpha$	Gini (wealth)	Gini (income)	Negative wealth [%]	Top 1% (wealth)	Top 1% (income)	Top 5% (wealth)	Top 10% (wealth)	Top 20% (wealth)	Top 30% (wealth)
<b>Data</b>	<b>0.790</b>	<b>0.470</b>	<b>11</b>	<b>30.0</b>		<b>51.0</b>	<b>64.0</b>	<b>79.0</b>	<b>88.0</b>
0.31	0.695	0.246	0	21.4	7.3	49.4	65.5	77.7	82.2
<b>0.36</b>	<b>0.724</b>	<b>0.287</b>	<b>0</b>	<b>22.6</b>	<b>8.7</b>	<b>51.0</b>	<b>67.4</b>	<b>80.6</b>	<b>84.8</b>
0.41	0.746	0.329	0	23.3	10.1	52.0	68.5	82.5	86.8
0.46	0.763	0.370	0	24.1	11.6	52.8	69.3	83.9	88.5
0.51	0.772	0.410	0	23.3	12.3	52.4	69.1	84.5	89.8

**Table 3.** Results for the calibration without debts.



## Conclusion

In the paper, we summarize the recent findings on the evolution of the factor shares. We analyse the consequences of the observed decline in the labour share for the income and wealth inequalities. We show that the decline in the labour share of one percentage point leads to the rise in the Gini coefficient of the income distribution of 0.008, on average. At the same time, the changes in the factor shares are almost neutral for the wealth inequalities.

The results have to be interpreted carefully, however, because our simple model does not account for few factors that may influence the findings. In particular, we assumed the constant labour supply, whereas the labour share decline is likely to affect the labour-related decisions of consumers. Moreover, we did not analyse the factors behind the movements of the factor shares and assumed the standard Cobb-Douglas technology with constant, exogenous factor shares. Although the causes of the factor share's instabilities have not been identified clearly yet, they are certainly important for the discussed topic.

Nonetheless, our results show that the changes in the factor shares play the important role in the evolution of the income inequalities, but seem less important as far as the wealth distribution is concerned. Further studies are definitely required to put more light on this topic.

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