Households investment portfolio performance evaluation

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Abstract

The main purpose of this paper is to present a theoretical discussion on performance evaluation of household investment portfolio. In order to do that, preferences of households need to be taken into account, which can be expressed by an appropriate utility function. Performance measures used in the market are strongly focused on linear models such as CAPM. It was there assumed that an increase of risk should be compensated by a proportional growth of expected return. Households, characterized by an increasing and concave utility function, expect a non-linear increase of the expected rate of return in exchange for the extra risk taken. It is important to find a performance measure that takes into account household indifference curves. For example, it might be the Generalized Sharpe Ratio (GSR), proposed by (Hodges, 1998), or a measure proposed by (Leland, 1999) modifying the traditional beta of CAPM so that it incorporates investor's utility function, that was suggested to replace Jensen's alpha.

Keywords: *performance measures, Sharpe ratio, utility function, household portfolio*
JEL Classification: G11, G14, D14

1. Introduction

The issue of investment performance evaluation, particularly for mutual funds and other forms of collective investing, has been a subject of research for many years. Also in Polish market the role of such types of financial institutions is important and still growing. Research on Polish market (NBP, 2013) performed for the end of 2012 showed that net assets of mutual funds amounted to 151.3 billion PLN, of which 72.5 billion belonged to households. This form of financial surplus allocation constituted 14.4% of all investments and deposits from households. Investments in risky funds, allocating their assets mainly in equity instruments, represent about 20% of all net assets of households. Performance of these funds has, thus, a very big impact on the value of household invested savings. This makes evaluating performance of portfolios managed by mutual funds a crucial issue, not only for households themselves, but also for national economy in general. Measuring performance might equip a household with tools for comparison between competing investments, or give grounds for a decision to change the fund managing institution. The need for such tools may be also substantiated by the fact that households realize their financial goals and in this process they

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often have to make choices between different investments. The choice of retirement investment is of particular significance (for more details comp., e.g., Feldman, Pietrzyk & Rokita, 2014).

In classical approaches to investment performance evaluation mainly linear models were used. It was there assumed that an increase of risk should be compensated by a proportional growth of expected return. In compliance with this approach Sharpe Ratio (Sharpe, 1966, 1994) became the basic performance indicator. It related the amount by which realized return exceeded the risk free rate to total risk taken on by the investment manager. A similar proposal presented (Treynor, 1965), who related the realized excess return to systematic risk. These solutions, however, did not take into account the fact that households are usually characterized by a concave utility function, which makes them require nonlinear increments of return for an increase in risk. It was pointed out by, amongst others, (Leland, 1999), who proposed replacing β coefficient of the linear model with a parameter adjusted to the utility function of an investor. As a result, also Jensen's alpha will be changed. This change will additionally allow for taking into account the asymmetry of systematic risk. Research in this direction was also done by Hodges (1998), who modified Sharp Ratio. Also (Stutzer, 2000) proposed a measure that was strictly related to the concept by Hodges, but its interpretation was less intuitive. Just another approach was proposed by Kaplan (2005), who named his indicator lambda, and built it on the basis of investor's optimum utility. Also a solution presented by (Pézier, 2008) is worth mentioning. It assumes performance evaluation by means of equivalent- of-certainty rate. A review of performance evaluation methods, including those based on utility, may be found, amongst others, in the research by (Cogneau & Hubner, 2009a, 2009b). Nonlinear models used for performance evaluation, besides aforementioned utility-based ones, encompass also a broad set of market-timing models, starting from Treynor-Mazuy (Treynor & Mazuy, 1966) and Henriksson-Merton (Henriksson & Merton, 1981). A research on performance of Polish mutual funds may be found, amongst others, in an article by (Pietrzyk, 2012)

The main aim of this paper is a theoretical discussion about portfolio management performance evaluation from the point of view of households, taking risk aversion into account, as well as presenting possibilities of its applications by the example of Polish mutual funds.

2. Evaluation of mutual fund performance

Using utility function in performance evaluation of mutual funds will require modification of one of the most popular performance measures, namely – Sharp Ratio (Sharpe, 1966, 1994). This measure takes into account both realized rate of return and total risk measured with standard deviation. Such a construction requires the assumption that rates of return are normally distributed. Sharp Ratio is given by the following formula:

$$Sh = \frac{R - R_f}{\sigma} \quad , \tag{1}$$

where R – mutual fund rate of return, R_f – risk-free rate of return, σ – standard deviation of returns.

Proposed by (Hodges, 1998) Generalized Sharpe Ratio (GSR) may be used in at least two cases. Firstly, when rates of return on a portfolio are not normally distributed, even if they are asymmetric. This allows to avoiding many paradoxes described in the literature (comp. (Hodges, 1998)). The second way it may be used is evaluation of investment performance from the point of view of an investor characterized by a utility function with a constant absolute risk aversion parameter.

For the needs of this article it is assumed that the investor maximizes expected utility by maximization of a utility function *E(U(w))*, which may take on in a particular case the following form: $U = -e^{-\lambda w}$, for any $\lambda > 0$, λ being the absolute risk aversion parameter.

Assuming that mutual funds available in the market show normal distributions of logarithmic returns with the distribution of the form: $N(\mu T;\sigma^2 T)$, and that utility function to be maximized is: $U = -e^{-\lambda w}$, the following equation is obtained, after substituting the term w with the formula describing the investment value:

$$\max_x E(U) = -e^{\left(-\lambda\left(\mu x T - \frac{1}{2}\lambda\sigma^2 T x^2\right)\right)}, \tag{2}$$

where μ – mean excess return, σ – standard deviation, T – length of the investment horizon in years, λ – absolute risk aversion parameter.

This function is to be maximized under the following condition: maximization of this function is the following: $\mu T - \lambda\sigma^2 T x = 0$, which, after transformation, gives: $x = \frac{\mu}{\lambda\sigma^2}$.

Substituting terms in the equation (2) it is obtained that:

$$U^* = \max_x E(U) = -e^{\left(-\frac{\mu^2}{2\sigma^2}T\right)}. \tag{3}$$

Using this equation a modified Sharpe Ratio estimate is obtained, in relation to Investor's optimal utility. Performance measure defined in this way may be also used if returns are not normally distributed. Hodges proposed a modification of Sharpe Ratio and expressing it by means of investor's utility, obtaining the Generalized Sharpe ratio, which takes on the following form:

$$GSR = \sqrt{\frac{-2}{T}\ln\left(-U^*\right)}, \tag{4}$$

where U^* – optimal expected utility.

This way of defining the measure with optimal utility *U**, which is independent of investor's risk aversion, boils down in fact to classical version of the Sharpe measure if distribution of return is normal. This is why it is worth modifying this approach by working into the model a relation between investor's risk aversion and the output of the model, that is – the conclusion about investment performance.

The investor, on the way to maximization of expected utility, will evaluate fund performance taking into account utility of each particular investment. Thus, maximization of expected utility for financial outcome yielded to the investor by mutual funds may be treated as maximization of performance measure of these funds from the point of view of this particular investor's risk aversion. The task discussed above may be simplified to the choice of the fund lying on the highest indifference curve. This boils down then to maximization of the following function:

$$CE = \mu - \frac{1}{2}\lambda\sigma^2, \tag{5}$$

where *CE* – certainty equivalent.

Thus, from amongst of the investments considered by an individual investor the one that offers the highest equivalent of certainty should be treated as the best.

The second proposition is to use the Generalized Share Ratio in such a way that the strength of investor's risk aversion is taken into account. To do this, instead of optimal utility, utility of ex-post results obtained by mutual funds will be used. Then, GSR takes on the following form:

$$GSR = \sqrt{\frac{-2}{T}\ln\left(-U\right)}, \tag{6}$$

where $U = -e^{\left(-\lambda\left(\mu T - \frac{1}{2}\lambda\sigma^2 T\right)\right)}$.

This way of Generalized Sharpe ratio defining allows to evaluate investment utility for mutual funds, as well as prepare their ranking based on historical data.

3. Empirical example

The survey of mutual fund performance covers 17 open funds whose excess rate of return over the risk free rate was positive in the period 2012-2013. The choice of this period was mainly motivated by availability of data for a big group of mutual funds, and also by the technical need of focusing on a period when the value of the analysed investments increased. It was also assumed that only the funds that invest in risky assets, and mainly stocks from Polish market, will be included into analysis. Benchmarks for these funds contain mainly WIG and WIG20 indices, and stock index weights in the benchmarks are at least 80% in all cases. The composition of benchmarks presents Table 1**.**

Fund	Symbol	Benchmark
Amplico Akcji Średnich Spółek	FIO1	90% mWIG40, 10% WIBID 1M
Amplico Subfundusz Akcji Polskich	FIO2	50% WIG20, 40% mWIG40, 10% WIBID 1M
Amplico Subfundusz Akcji	FIO3	70% WIG20, 30% mWIG40
Arka BZ WBK Akcji	FIO4	75% WIG + 20% [(MSCI Emerging Europe ex Russia) + Austria Index] + 5% WIBID O/N
Aviva Investors Polskich Akcji	FIO5	90% WIG, 10% Citigroup PLN 1M Eurodeposit Local Curency
BPH Akcji	FIO6	95% WIG, 5% WIBID 3M
ING SFIO Akcji 2	FIO7	80% WIG, 20% WIBID O/N
ING Subfundusz Akcji	FIO8	100% WIG
Investor Akcji Dużych Spółek	FIO9	80% WIG20, 10% BUX, 5% PX, 5% WIBID 6M
Investor Akcji	FIO10	90% WIG, 10% WIBID 6M
Legg Mason Subfundusz Akcji	FIO11	100% WIG
Noble Fund Akcji	FIO12	90% WIG, 10% WIBID O/N
PKO Akcji	FIO13	85% WIG, 15% WIBID O/N
PZU Akcji Krakowiak	FIO14	90% WIG20, 10% WIBID 3M
Skarbiec Akcja	FIO15	90% WIG20, 10% WIBID 3M
Pioneer Akcji Polskich	FIO16	100% WIG
UniKorona Akcje	FIO17	100% WIG

Table 1 Mutual funds and their benchmarks.

In the period spanned by the research the analysed funds yielded positive rates of return (Table 2). Only for one of the funds gave a negative rate of return. The investment outcomes of the mutual funds are also compared with basic indices of Polish market, WIG and WIG20.

Period	FIO1	FIO2	FIO3	FIO4	FIO5	FIO6	FIO7	FIO8	FIO9	FIO10
2012	13.6%	14.0%	16.0%	19.8%	20.9%	20.1%	19.9%	23.2%	16.2%	20.4%
2013	18.8%	4.9%	1.0%	1.5%	15.1%	5.5%	9.1%	5.7%	3.6%	20.4%
2012-2013	32.3%	18.9%	17.0%	21.3%	36.0%	25.6%	29.0%	28.9%	19.8%	40.8%
Period	**FIO11**	**FIO12**	**FIO13**	**FIO14**	**FIO15**	**FIO16**	**FIO17**	**WIG20**	**WIG**	
2012	15.6%	22.8%	17.8%	18.6%	22.8%	14.8%	19.0%	18.6%	23.3%	
2013	12.1%	7.9%	9.9%	7.3%	4.6%	6.0%	3.9%	-7.3%	7.8%	
2012-2013	27.8%	30.7%	27.7%	25.9%	27.4%	20.8%	22.9%	11.3%	31.0%	

Table 2 Mutual funds and indices rate of returns.

The results have been compared with risk free rates. Interbank WIBOR 12M rate was used as an approximate of the risk free rate. At start, on Dec. 31, 2011, it was at the level of 4.9%, and on Dec. 31, 2012, it was 3.88%. In this basis realized excess returns (over the risk free rate) were determined for all funds, as well as for WIG and WIG20 indices. Three mutual funds turned out to yield negative excess returns.

The research on mutual fund performance are done for three periods: two annual ones (i.e., 2012 and 2013) and the whole period 2012-2013. Sharpe Ratios and GSRs were calculated for all 17 funds, as well as WIG and WIG20 indices. These measures were calculated for average daily logarithmic returns in the aforementioned periods. Measures for the whole period 2012-2013 are presented in

Period	Measure	FIO11	FIO12	FIO13	FIO14	FIO15	FIO16	FIO17	WIG20	WIG
2012-2013	Sharpe	0.049	0.055	0.052	0.038	0.039	0.027	0.034	0.005	0.047
	GSR $\lambda = 1$	0.026	0.028	0.026	0.024	0.025	0.020	0.022		0.028
	GSR $\lambda = 2$	0.035	0.038	0.036	0.032	0.033	0.025	0.029		0.037
	GSR $\lambda = 5$	0.048	0.052	0.049	0.038	0.038	0.021	0.034		0.047
	GSR $\lambda = 8$	0.048	0.054	0.052	0.022	0.013		0.017		0.039
	GSR $\lambda = 10$	0.042	0.050	0.049						0.015
	GSR $\lambda = 12$	0.027	0.039	0.041						

GSR λ= 15

Table 3.

Period	Measure	FIO1	FIO2	FIO3	FIO4	FIO5	FIO6	FIO7	FIO8	FIO9	FIO10
2012-2013	Sharpe	0.057	0.025	0.018	0.030	0.075	0.038	0.051	0.046	0.026	0.087
	GSR λ= 1	0.029	0.018	0.016	0.021	0.032	0.024	0.027	0.027	0.019	0.034
	GSR λ= 2	0.040	0.023	0.018	0.027	0.044	0.032	0.037	0.036	0.024	0.048
	GSR λ= 5	0.054	0.020		0.028	0.063	0.037	0.050	0.046	0.022	0.070
	GSR λ= 8	0.057				0.073	0.020	0.050	0.040		0.081
	GSR λ= 10	0.053				0.075		0.045	0.024		0.085
	GSR λ= 12	0.042				0.075		0.032			0.087
	GSR λ= 15					0.069					0.084

Period	Measure	FIO11	FIO12	FIO13	FIO14	FIO15	FIO16	FIO17	WIG20	WIG
2012-2013	Sharpe	0.049	0.055	0.052	0.038	0.039	0.027	0.034	0.005	0.047
	GSR λ= 1	0.026	0.028	0.026	0.024	0.025	0.020	0.022		0.028
	GSR λ= 2	0.035	0.038	0.036	0.032	0.033	0.025	0.029		0.037
	GSR λ= 5	0.048	0.052	0.049	0.038	0.038	0.021	0.034		0.047
	GSR λ= 8	0.048	0.054	0.052	0.022	0.013		0.017		0.039
	GSR λ= 10	0.042	0.050	0.049						0.015
	GSR λ= 12	0.027	0.039	0.041						
	GSR λ= 15									

Table 3 Mutual funds' performance measures for different level of λ.

Sharpe Ratio took on a negative value for 4 funds in 2013. This is why it was not determined in this period for these funds. Generalized Sharpe Ratio was not defined for many funds. The number of funds without a positive GSR increases with the growth of investor risk aversion. For $\lambda = 5$ there were no such funds in 2012, 11 funds in 2013 and 1 fund in the period of 2012-2013. For $\lambda = 15$ this was, however, 7 funds in 2012 and 17 funds in 2013 and 17 in the period 2012-2013.

The next step of the research was preparing rankings of mutual fund performance on the basis of the obtained Sharpe Ratio and Generalized Sharpe Ratio estimates. These rankings were constructed for the three periods described above and for a number of different absolute risk aversion levels. The results presented in the Table 4-5 show that performance rankings based on these two indicators may differ significantly. These differences are most evident in the year 2012, when realized rates of returns were the highest.

Fund	**SR**	**GSR**						
		λ=1	**λ=2**	**λ=5**	**λ=8**	**λ=10**	**λ=12**	**λ=15**
FIO10	1	1	1	1	1	1	1	1
FIO5	2	2	2	2	2	2	2	2
FIO1	3	3	3	3	3	3	3	---
FIO12	4	4	4	4	4	4	5	---
FIO13	5	8	8	6	5	5	4	---
FIO8	6	6	6	5	6	6	6	---
FIO11	7	9	9	7	7	7	7	---
WIG	8	5	5	8	9	9	---	---
FIO7	9	7	7	9	8	8	---	---
FIO15	10	10	10	11	13	---	---	---
FIO14	11	11	11	10	10	---	---	---
FIO6	12	12	12	12	11	---	---	---
FIO17	13	13	13	13	12	---	---	---
FIO4	14	14	14	14	---	---	---	---
FIO16	15	15	15	16	---	---	---	---
FIO9	16	16	16	15	---	---	---	---
FIO2	17	17	17	17	---	---	---	---
FIO3	18	18	18	---	---	---	---	---

WIG20	19	---	---	---	---	---	---	---

Table 4 Ranking of mutual funds and indices in years 2012-2013 by Sharpe Ratio (SR) and GSR for different level of λ.

A problem with the use of GSR arises when returns of the funds in question are only slightly higher than the risk free rate (when the risk is high at the same time, which is an often situation in the case of stocks funds), and the risk aversion level of the investor is high. Then, GSR is undefined and it is impossible to evaluate the funds using this measure.

	2012								**2013**							
Fund	**SR**	**GSR**							**SR**	**GSR**						
	λ=	1	2	5	8	10	12	15		1	2	5	8	10	12	15
FIO5	1	5	5	4	2	2	2	1	1	1	1	1	1	1	1	1
FIO12	2	3	2	1	1	1	1	2	2	2	2	2	2	2	3	---
FIO7	3	1	1	2	3	3	3	3	3	3	3	3	3	3	2	2
WIG	4	2	3	3	4	4	5	6	4	4	4	4	4	4	---	---
FIO10	5	6	6	6	5	5	4	4	5	5	5	5	5	---	---	---
FIO15	6	4	4	5	6	7	9	10	6	6	6	6	---	---	---	---
FIO8	7	7	7	7	7	6	6	5	7	7	7	---	---	---	---	---
FIO4	8	9	8	8	8	8	7	7	8	8	8	---	---	---	---	---
FIO17	9	10	10	9	9	9	8	9	9	9	9	---	---	---	---	---
FIO13	10	13	12	12	10	10	10	8	10	10	10	---	---	---	---	---
FIO14	11	11	11	11	11	11	11	11	11	11	---	---	---	---	---	---
FIO6	12	8	9	10	12	12	12	---	12	12	---	---	---	---	---	---
FI011	13	16	15	13	13	13	13	12	13	13	---	---	---	---	---	---
FIO9	14	14	14	15	16	17	17	---	14	---	---	---	---	---	---	---
FIO3	15	15	16	16	15	16	16	---	15	---	---	---	---	---	---	---
WIG20	16	12	13	14	18	19	---	---	---	---	---	---	---	---	---	---
FIO1	17	19	19	17	14	14	14	---	---	---	---	---	---	---	---	---
FIO2	18	18	18	18	17	15	15	---	---	---	---	---	---	---	---	---
FIO16	19	17	17	19	19	18	---	---	---	---	---	---	---	---	---	---

Table 5 Ranking of mutual funds and indices in year 2012 and 2013 by SR and GSR.

Conclusions

The research presented here indicate that preferences of an investor may significantly influence results of portfolio management performance evaluation and classical measures, like Sharpe Ratio, do not reflect the requirements by the households as to the additional rate of return compensating an additional portion of investment risk. The approach presented in this article shows a way of evaluating mutual fund performance allowing to overcome some drawbacks of the classical approach. Put it more generally, the concept discussed here may be useful in evaluation of portfolio management performance, taking into account individual attitude towards risk in financial markets.

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