An application of the harmonic oscillator mechanism in the verification of the theory of economic growth by Dunning based on the examples of selected countries of Central and Eastern Europe

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Abstract

Analogies with mechanisms ruling the natural world have often been sought in the course of economic phenomena. This paper is also an attempt at combining the physical phenomenon of a harmonious oscillator with the theory of economic growth by Dunning [2]. In his theory, Dunning distinguished stages of economic growth of countries that imply the dependency between the investment position of countries and their GDP *per capita*, while the graph presenting this dependency reminds a trajectory of oscillating motion of a damped harmonic oscillator. This analogy has given inspiration to reinterpret the theory of economy on the grounds of the mechanism of a physical model. In this paper, the harmonious oscillator motion equation was adapted to the description of dependencies shown in the theory of economic growth by Dunning. The mathematical solution of this equation is properly parameterised and parameters are estimated with the use of the Gauss-Newton algorithm. The main objective of this paper is to allocate a specific stage in the economic growth to each country on the basis of the values of parameter estimations of the proposed cyclical models of changes in the net investment indicator.

Keywords: *Dunning's theory, Gauss-Newton algorithm*

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1. Introduction

Dependencies between the country's economic growth and the level of inward and outward foreign direct investments are the subject of numerous research works. There is a range of economic theories which attempt to describe such dependencies. One of these theories which is really worth attention is the theory of economic growth formulated by Dunning [2]. In his theory, Dunning distinguished stages of economic growth that imply the dependency between the investment position of countries (NOI *per capita*[2]) and their GDP *per capita*, while the graph presenting this dependency reminds a trajectory of oscillating motion of a damped harmonic oscillator. This analogy has inspired the author of this article to reinterpret the

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[2] Net outward investment (NOI) is the difference between foreign investments located outside the borders of a given country by companies based in this country and foreign investments executed by foreign companies within the territory of this country [2].

theory of economy on the grounds of the mechanism of the operation of a physical model. In this article, the harmonic oscillator motion equation was adapted to describe the dependencies presented in Dunning's economic growth theory. Through the proper parameterisation, the proposed econometric model describes the path of the country's investment development in the dynamic perspective, which allows for allocating the country to the relevant stage of economic growth distinguished in Dunning's theory. An advantage of this model lies also in its capacity to detect convergence or divergence phenomena in the country's investment development. The application of the harmonic oscillator model in describing economic phenomena (especially the modelling of economic development) is known in the theory of economics (cf. e.g. [1], [5], [8], [9]) but few researchers were able to use it in the empirical verification of Dunning's theory of economic growth [3]. The elementary objective of this article is to allocate the relevant stages in the economic growth to the examined countries on the basis of the estimated parameters of the proposed model of cyclical changes in the net outward investment index. The research covered 10 selected countries from Central and Eastern Europe being EU members. Eurostat data for the period 1995-2012 was used in calculations.

2. Modelling the path of the country's investment development

In his theory, Dunning [2] distinguished five stages of the country's economic growth, which are strictly connected with the level of GDP *per capita* as well as values and directions of changes in NOI *per capita*. These stages occur consecutively, creating a sort of an investment development path.

The stages of economic growth distinguished by Dunning may be characterised as follows [2]:

- Countries with the weakest economy with GDP *per capita* below USD 400[3] in which the value of NOI *per capita* is close to zero and negative undergo the first stage of development. These countries are characterised by the deficit of capital, so they do not export foreign direct investments themselves but at the same time do not attract foreign capital, either;
- The second stage of growth involves countries with GDP *per capita* from the range of USD 400-1500 and is characterised by the negative value of NOI *per capita*. Countries being in this stage of growth are to a certain extent more attractive to foreign investors but still have too low supply of capital to export foreign direct investments;

[3] Boundary values of GDP *per capita* specified by Dunning are no longer valid (they were determined on the basis of data for the period 1967-1978). However, the very mechanism of identifying the stages of economic growth is constantly used in research works devoted to economic growth and in reports of institutions monitoring economic growth (e.g. UNCTAD).

- The third stage of growth involves countries with GDP *per capita* from the range of USD 2000-4750 having a negative but increasing value of NOI *per capita*. These countries conduct their own foreign direct investments more and more often but the scale of these investments is lower than inward foreign direct investments in these countries;
- The fourth and fifth stage involves highly developed countries with GDP *per capita* above USD 2600 in which NOI *per capita* is usually positive, which shows that these countries are active exporters of foreign direct investments.

The course of the curve illustrating the dependence of NOI *per capita* on the value of GDP *per capita* reminds the oscillating motion of a damped harmonic oscillator. In general, the oscillator motion equation takes the following form [6]:

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \varpi_0^2 x = 0, \tag{1}$$

where: x – displacement of the oscillator from the equilibrium position, t – time, β – elasticity index, ω_0 – frequency of free vibrations.

Equation (1) is commonly used to describe the motion of a mechanical oscillator or an electromagnetic oscillator dampened by external factors (forces of resistance, friction, etc.). Equation (1) is a homogeneous second-order differential equation the solution of which in case of weak dampening (i.e. when $\beta < \omega_0$) takes the form:

$$x = x_0 e^{-\beta t}\cos(\omega t + \varphi), \tag{2}$$

where: $\omega = \sqrt{\omega_0^2 - \beta^2}$, φ – phase shift, x_0 – initial displacement of the oscillator.

If one assumes that values of NOI *per capita* change approximately according to equation (1), then with the use of function (2) one can formulate the following model describing cyclical changes of this index:

$$NOI_t = \alpha e^{-\beta PKB_t}\cos(\gamma PKB_t) + \varepsilon_t. \tag{3}$$

In model (3) there are three parameters requiring estimation: α, β, γ, that characterise the course of dependence between NOI and GDP in the following manner:

1. Parameter α determines the initial value of amplitude (with $\beta = 0$) and the starting direction of changes in NOI (increase towards the positive orientation of the vertical axis of the Cartesian coordinate system, when α>0 or decrease towards the negative orientation of this axis when α<0). The value of parameter α estimator may also suggest the capacity of the

economy to absorb foreign direct investments or export own foreign direct investments. High and negative estimation value of parameter α estimator may suggest strong investment competition of the economy of a given country which is effective in attracting foreign capital. High and positive estimation value of this parameter, in turn, may indicate a high economic potential (with considerable supply of capital) of the country actively conducting foreign direct investments.

2. Parameter β determines the intensity of vibration dampening. Depending on the sign of parameter β, displacements from the equilibrium position can have threefold character:

- $\beta > 0$ indicates decreasing displacements,
- $\beta < 0$ indicates increasing displacements,
- $\beta = 0$ indicates the constant amplitude of displacements.

The positive value of parameter β can indicate symptoms of convergence[4] which results in the economy of a given country becoming similar to the economies of highly developed countries in the range of the pattern of the investment development path. The negative value of parameter β may be interpreted as the occurrence of a phenomenon opposite to convergence, i.e. divergence, which results in greater distance in the investment development in relation to the economies of developed countries.

3. Parameter γ indicates the length of one cycle in oscillatory motion (distance between two successive upper or lower turning points), whereas the full cycle period is $2\pi\gamma^{-1}$. A longer cycle period is characteristic of better developed economies while the shorter one occurs in general in economies at a weaker level of development.

The analysis of a specific configuration of the values of estimations of the discussed parameters makes it possible to locate a considered country in one of the stages of economic growth defined by Dunning. The expected stages in the development of countries are presented in chart 1, depending on the estimators of parameters α and β.

When analysing the content of table 1, it must be stressed that if parameter α does not differ considerably from zero, then the country may be in the first stage of development or at the beginning of the fourth stage (the amplitude of oscillation is inconsiderable then). In this situation, the familiarity with the sign of parameter α estimator can be helpful in classifying a country into one of the stages: a positive sign indicates the fourth stage while the negative one suggests the first stage.

[4] Convergence involves the penetration of economy patterns of countries with a similar level of wealth and differentiation of patterns in countries with different wealth levels (cf. e.g. [4], [7]).

Estimator signs		Stage of economic growth
α	β	
0*)	any	1 or 4
–	–	2 or 3
–	+	3
+	–	4
+	+	5

Note: *) Zero value is to be understood as a statistically insignificant result.

Table 1 Stages of economic growth depending on the signs of parameters α and β in model (3).

It must be also stated that the classification of countries into the second or third stage may not be strict (in case $\alpha<0$ and $\beta<0$). In that case, when allocating to one of the stages, one can also use the estimator of parameter γ which – if known – enables the calculation of the length of the cycle ($2\pi\gamma^{-1}$). The results of empirical research confirm that a higher level of GDP is associated with a longer stage of the cycle (so a lower value of the estimator of parameter γ) [3]. Low and statistically significant values of the estimators of parameter γ may suggest a higher level of the country's economic growth.

3. Results of the estimation of the harmonic oscillator model for the countries of Central and Eastern Europe

The parameters of model (3) were estimated for ten countries of Central and Eastern Europe being member states of the European Union on the basis of the data for the period 1995-2012, with the use of the non-linear method of least squares – the Gauss–Newton algorithm. The results of the estimation of the parameters of model (3) are presented in table 2 (in brackets under estimations of parameters test probabilities *p* are to be found). The chart also contains values of the model's coefficients of determination and numbers of stages in the economic growth ordered for individual countries on the basis of the values of parameter estimations.

When analysing the results from table 2, it must be stated that some results are not statistically significant, therefore despite the generally good match of the estimated model (3) to empirical data, conclusions about the unambiguous allocation of individual countries to specific stages in the development must be formulated carefully in some cases.

On the basis of the results contained in table 2, it seems that the highest fourth stage of the economic growth was achieved by Slovakia. But this classification must be treated with caution as parameters α and γ were not statistically significant in model (3) estimated for Slovakia and the very model is adjusted rather moderately well to statistical data.

Country	Parameter α	Parameter β	Parameter γ	Coefficient of determination R^2 (%)	Stage of development
Bulgaria	-10.898 (**0.030**)	-0.002 (0.982)	0.000 (0.995)	55.47	2-3
Czech Republic	-3.372 (**0.035**)	0.002 (**0.021**)	0.154 (0.310)	89.21	3
Estonia	-8.191 (**0.005**)	-0.004 (0.083)	0.000 (0.954)	60.47	2-3
Hungary	-2.067 (**0.014**)	-0.005 (**0.001**)	0.000 (0.894)	80.11	2-3
Latvia	-3.946 (0.977)	-0.000 (0.998)	0.240 (**0.000**)	48.72	1
Lithuania	-3.487 (**0.008**)	-0.004 (**0.025**)	0.000 (0.947)	83.47	2-3
Poland	-15.201 (**0.043**)	0.011 (0.186)	0.110 (**0.000**)	75.19	3
Romania*)	5.621 (0.803)	0.001 (0.957)	0.153 (0.109)	---	---
Slovakia	1958.736 (0.667)	-0.005 (**0.000**)	0.000 (0.934)	49.55	4
Slovenia	-156.906 (**0.028**)	0.000 (**0.012**)	0.000 (**0.048**)	90.97	3

Note: *) Stable estimations of parameters with the use of the Gauss-Newton algorithm were not obtained for Romania. The value p-values lower than 0.05 bolded.

Table 2 Results of estimation of the parameters of model (3) for the countries of Eastern and Central Europe.

Czech Republic, Poland and Slovenia may be classified in the third stage of economic growth. But while in case of Czech Republic and Slovenia, parameter β turned out to be statistically significant (at the significance level of 0.05), then in case of Poland this parameter is not significant, so the classification of Poland to the third stage of growth must be done with certain caution.

The results of the estimation of parameters in model (3) most frequently indicated the second or third stage of growth in the examined countries. This situation occurred in case of Bulgaria, Estonia, Lithuania and Hungary. It should also be pointed out that model (3) estimated only for Lithuania and Hungary had both parameters α and β statistically significant and in relation to these countries the conclusion concerning the classification to the second or third stage seems particularly "strong". However, parameter γ estimated for these countries was not statistically significant, which makes it impossible to determine the length of the cycle stage in a reliable way and to allocate each of these countries to one of two stages (second or third) in an unambiguous way.

The lowest level of growth was allocated to Latvia (first stage of growth) but here it must be emphasised that key parameters decisive for categorising the country to a specific stage of growth are not statistically significant. Stable estimations of the parameters of model (3) were not obtained only in case of Romania, so it was not possible to allocate this country to the relevant stage of economic growth.

Parameter β achieved a positive and statistically significant value in model (3) estimated for the Czech Republic and Slovenia. This suggests the possibility of occurrence of clear symptoms of decreasing proportions in the investment development in these countries in relation to the majority of "old" EU countries. In the model estimated for Poland, parameter β is positive (and may suggest convergence) but statistically insignificant. In case of other countries, the negative value of the parameter β estimators suggests symptoms of divergence, i.e. the possibility of increasing distance in the investment development when compared to economically developed countries. However, only in model (3) built for Lithuania, Slovakia and Hungary, parameter β turned out to be negative and statistically significant, which suggests that divergence may be distinct there.

4. Conclusions

The approach to the examination of relations between the level of NOI *per capita* and the level of GDP *per capita* proposed in this article in the context of the verification of Dunning's theory of economic growth seems to be an interesting alternative for the most frequently applied solutions in this field. So far, the most frequent method of examining the investment development path involves the polynomial models of regression describing the dependence between NOI and GDP. Even though they enable the approximation of the examined dependence, they also have several important limitations. First of all, it is difficult to give substantive interpretation to parameters in such models, which does not allow for specifying unambiguously the stages of the country's economic development and for stating whether an investment development path expires or develops.

The proposed model using the operating mechanism of a harmonious oscillator seems to cope well with these difficulties. An elementary advantage of this model is an easy interpretation of its parameters, which enables the creation of simple rules of allocating a country to a specific stage of economic growth. The proposed approach makes it also possible to model the path of investment development in the dynamic perspective and by analysing its fluctuations allows for evaluating the degree of convergence or divergence as well as the length of the very investment cycle. Classical polynomial regression models did not give such possibilities in the range of the analysis of the investment development path.

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