Selected methods of choice of the optimal monetary policy transmission horizon

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Abstract

Analyzing the responses of economic variables, including inflation to the change of the level of monetary policy instrument, we can take into consideration the forecast of economic variables in appropriate horizon, at least equal to the monetary policy transmission horizon. These forecasts are the operational objective when we make the decisions for inflation forecast targeting. When we realize the direct inflation targeting strategy, the inflation forecasts are important. In this paper we present the selected methods of choice of the optimal monetary policy transmission horizon in inflation target realization context. We determine the optimal horizon using the traditional reduced rank vector autoregression model of monetary policy for inflation forecasting.

Keywords *optimal monetary policy transmission horizon, reduced rank VAR model, generalized impulse response*

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1. Introduction

National Bank of Poland and many other central banks including the European Central Bank pursue an explicit inflation target. Accordingly the important problem is choice of the optimal monetary policy horizon that is the appropriate number of periods ahead when the operational target is achieved. Note the operational target is achieved inflation rate k periods ahead equals inflation target.

Therefore the optimal horizon can be the time – number of periods ahead, at which inflation should be on target in the future assuming a shock occurs today, while the authorities determined the policy instrument minimizing their loss function.

In the case of the direct inflation targeting strategy, as an operational objective we require sometimes achievement of the inflation rate in the monetary policy transmission horizon belonging to a certain interval.

According to Rudebusch and Svensson's interpretation of inflation targeting we need solve the optimisation problem that minimalizes deviations of inflation from target and of output from potential output.

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At the beginning we will discuss the theoretical foundations of the reduced rank vector autoregression model.

2. Reduced rank vector autoregression model - theoretical foundations

To explore the optimal horizon issue, we use the traditional reduced rank vector autoregression model for monetary policy.

Since between inflation and monetary policy instruments and other macroeconomic variables is feedback, that is, they are jointly correlated nature, to modelling of inflation can be used vector autoregression models, including reduced rank vector autoregressive models. Overall, generally reduced form of VAR model of order p without constant term can be written in matrix form as follows:

$$y_t = A_1 \cdot y_{t-1} + A_2 \cdot y_{t-2} + ... + A_p \cdot y_{t-p} + e_t \qquad (1)$$

where:

y_t - vector of all variables in the model at time *t*;

y_{t-i} - vector of all variables in the model at time *t*- *i*, for *i* = 1, 2,…, *p*;

A_i - matrices of coefficients, $i = 1,2,..., p$;

e_t - vector whose coordinates are the shocks that are uncorrelated with each other and they are the white noise. The variance and covariance matrix Ω of vector ε_t is a diagonal matrix.

Writing a model (1) in the following matrix form:

$$y_t = A \cdot X_t + e_t \qquad (2)$$

where y_t - vector of all variables in the model at time *t*; $X_t = [y_{t-1} \quad y_{t-2} \quad \cdots \quad y_{t-p}]^T$; $A = [A_1 \quad A_2 \quad \dots \quad A_p]$ and denoting by $A(L)$ the lag operator applied to the matrix *A* and the identity matrix by *I*, model (2) can be written in the equivalent form as follows:

$$(I - A(L))y_t = e_t \qquad (3)$$

in which $A(L) = A_1 \cdot L + A_2 \cdot L^2 + ... + A_p \cdot L^p$,

therefore $I - A(L) = I - A_1 \cdot L - A_2 \cdot L^2 - ... - A_p \cdot L^p$.

Lag operator can be written in general form as:

$$A(L) = B(L) \cdot C(L) \tag{4}$$

where: $B(L) = B_0 + B_1 L + B_2 L^2 + \ldots + B_q L^q$, $C(L) = C_1 L + C_2 L^2 + \ldots + C_p L^p$.

If q = 0, the model is called the reduced rank vector autoregressive model – RR-VAR model. Then the lag operator has the form $A(L) = B_0 C_1 L + B_0 C_2 L^2 + \ldots + B_0 C_p L^p$.

Assuming that $B = B_0$, $C = [C_1 \quad C_2 \quad \ldots \quad C_p]$, the matrix A of the reduced rank vector autoregressive model can be written as $A = BC'^T$, where: matrix B has dimension $k \times r$, and $C^T = [C_1 \quad C_2 \quad \ldots \quad C_p]^T$ has dimension $r \times k \cdot p$, k – number of variables in the model, p – vector autoregression rank, r is cointegration rank.

Least squares estimators of parameters of RR-VAR model are calculated from the formula $\tilde{A} = \tilde{B} \cdot \tilde{C}'^T$ in which [5]:

$$\tilde{B} = \Sigma_u^{\frac{1}{2}} \cdot \tilde{V} \tag{5}$$

$$\tilde{C}^T = \tilde{V}^T \Sigma_u^{-\frac{1}{2}} Y X^T (X X^T)^{-1} \tag{6}$$

assuming that Y – observation matrix of y_t, X – observation matrix of $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$, for $t = p+1, p+2, \ldots, N$, N - number of observations (length of the sample).

In the formulas (5) i (6) $\tilde{V} = [\tilde{v}_1 \quad \tilde{v}_2 \quad \ldots \quad \tilde{v}_r]$ is the matrix of the orthonormal eigenvectors corresponding to the r the largest eigenvalues of the following matrix:

$$\frac{1}{N-p} \Sigma_u^{-\frac{1}{2}} Y X^T (X X^T)^{-1} X Y^T \Sigma_u^{-\frac{1}{2}} \tag{7}$$

Furthermore, since Σ_u is any positive definite matrix, it can be assumed that

$$\tilde{\Sigma}_u = \frac{1}{N-p} Y (I_{N-p} - X^T (X X^T)^{-1} X) Y^T \tag{8}$$

where: I_{N-p} is the $(N-p) \times (N-p)$ identity matrix.

Since the analysis of the impact the monetary decision on inflation was based on the traditional reduced rank vector autoregressive model for monetary policy than we present the general form of this model below.

In traditional VAR models of monetary policy we select three variables: the inflation rate, the interest rate (eg. the reference rate) and the production – the output dynamics - for monthly data and the GDP - for quarterly data. Then in model (2) we have $y_t = [\pi_t \quad Y_t \quad i_t]^T$; $e_t = [e_{1t} \quad e_{2t} \quad e_{3t}]^T$ where π_t - inflation rate at time *t*, i_t - reference rate at time *t*, Y_t - output at time *t*; e_{1t} - inflation shock, e_{2t} - output shock, e_{3t} - interest rate shock. Vector e_t of white noise shocks; it is assumed that e_t is the three-dimensional random variable having a normal distribution $N(\theta, D)$, where *D* – variance and covariance matrix.

3. Optimalization problem

We assume that policy-makers wish to minimize the intertemporal loss function, that is, decision-makers should take into consideration the solution of following optimisation problem [3]:

$$E_t \sum_{k=0}^{\infty} \delta^k \cdot L_{t+k} \to \min \tag{9}$$

Constraints in the minimalization problem are the equation of traditional VAR models of monetary policy.

In the problem (9) $E_t \sum_{k=0}^{\infty} \delta^k \cdot L_{t+k}$ is intertemporal loss function at period *t*, δ is discount factor, $0 < \delta < 1$, E_t is symbol of the expected value determined on the basis of information available at the period t, L_t is temporal loss function.

The temporal loss function can take various forms. One of form of temporal loss function is quadratic loss function. This function is following:

$$L_t = \lambda_\pi \cdot (\pi_t - \pi^*)^2 + \lambda_Y \ (Y_t - Y_t^*)^2 + \lambda_{\Delta i} \cdot (\Delta i_t)^2 \tag{10}$$

where π_t is inflation rate, π^* is the inflation target, Y_t is log output, Y_t^* is potential log output, $\lambda_\pi, \lambda_Y, \lambda_{\Delta i}$ - the weights assigned to deviations of inflation from target, deviations of log output from potential log output and volatility in the first difference of the nominal interest rate, respectively.

4. Optimal transmission monetary policy horizon

We have two operational definitions of optimal transmission monetary policy horizon: an absolute and a relative horizon concept [1]. Taking into consideration our discussion in introduction, we present the first definition. We define the optimal transmission monetary policy horizon as the time at which it is least costly, for a given loss function, to bring inflation back to target after a shock. Operationally, this horizon is given by the number of periods after a shock when inflation is back on target under an optimal rule.

Because since 2004 the Polish National Bank pursues continuous inflation target of 2.5% with a maximum deviation of 1 percentage point up or down, we consider optimal transmission monetary policy horizon as referring to target range, so it can also determine optimal horizon as the number of periods ahead *k*, at which inflation has returned permanently to within a target or target range, following a shock today.

Since optimal horizon will vary according to the nature of the economic shock, we compute optimal horizon under the first criterion for different kinds of shocks.

5. The generalized impulse response GIR

The most intuitive tool to analysis the impact shock on the other variables in the system is the impulse response function. We have the different methods for identifying the shock response. One of this methods presented in this paper is the generalized impulse response GIR

The method of generalized impulse response has been proposed by the Koopa, Pesarana and Potter [2]. This method involves comparing two forecasts of the model. One forecast takes into account one-time shock, while the second forecast is determined for the situation without the occurrence of shock. Thus, the generalized impulse response $GIR_y(n, e_j, w_{t-1})$ is the difference of two conditional expected values, which can generally write for the vector y as follows:

$$GIR_y(n, e_j, w_{t-1}) = E(y_{t+n} / e_j, w_{t-1}) - E(y_{t+n} / w_{t-1}) \tag{11}$$

where y_{t+n} – vector variables of the model at time $t+n$, n – horizon of analysis, e_j – shock vector that corresponds to $k \times 1$ vector with not null element at the *j*- th element and zeros elsewhere, w_{t-1} – historical or starting values of the variables in the model, $E(\cdot/\cdot)$ – the conditional expected value.

When we determine the generalized response of inflation to shocks we take into consideration the first coordinate of the vector $GIR_y(n, e_t, w_{t-1})$.

Assuming the residuals from the VAR model are multivariate normally distributed, we have that the generalized impulse response from a shock (one standard deviation) to the *j*- th residual is given by

$$GIR_y(n, e_j, w_{t-1}) = \frac{1}{\sqrt{\sigma_j^2}} A^n \cdot D \cdot e_j \,. \qquad (12)$$

The matrix *A* is a matrix associated with the operator $A(L) = A_1 \cdot L + A_2 \cdot L^2 + ... + A_p \cdot L^p$, then $A = A_1 + A_2 + ... + A_p$.

6. The empirical analysis

For the calculation of optimal monetary policy transmission horizon we use the monthly inflation rate data (data published by Central Statistical Office) (source: www.stat.gov.pl) the monthly reference rate data (data published by the NBP) - data at the end of the month, as well as monthly industrial production growth rate data (data published by the Central Statistical Office) (source: www.stat.gov.pl) from the period January 2004 to March 2010 year.

Figures 1 to 3 show the generalized response of inflation associated with presented VAR model to different values (0.25%, 0.5%, 1%) of shocks: inflation shock, output shock and interest rate shock respectively.

The following Table 1 summarizes the optimal monetary policy transmission horizons for different types and values of shocks. This horizon determine on the basis of presented VAR model.

Based on the analysis it can be seen that the optimal monetary policy transmission horizon is primarily dependent on the size of the shock, rather than its type. The value of shock is higher the optimal horizon is greater.

Type of shock	Value of shock		
	0.25%	0.5%	1%
Inflation shock	2	4	7
Output shock	2	5	7
Interest rate shock	2	4	7

Table 1 The optimal monetary policy transmission horizons.

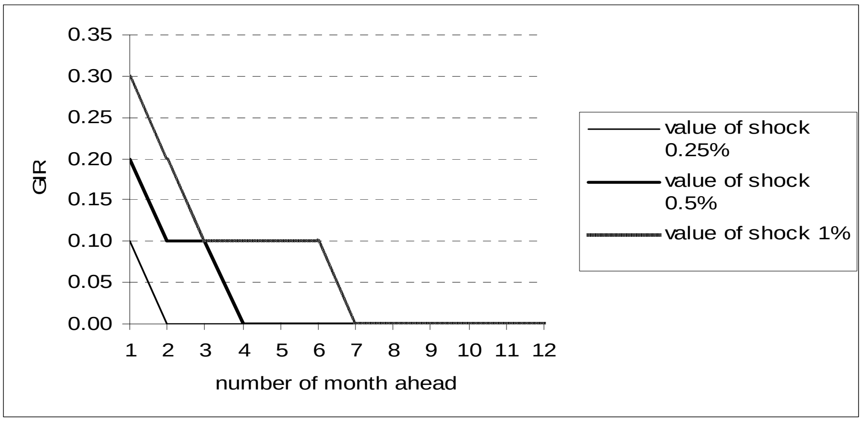

Fig. 1. The generalized impulse response of inflation in the face of inflation shocks.

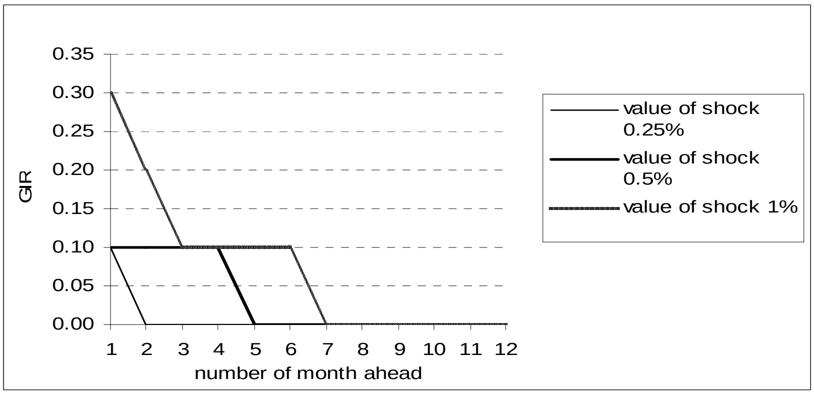

Fig. 2. The generalized impulse response of inflation in the face of output shocks.

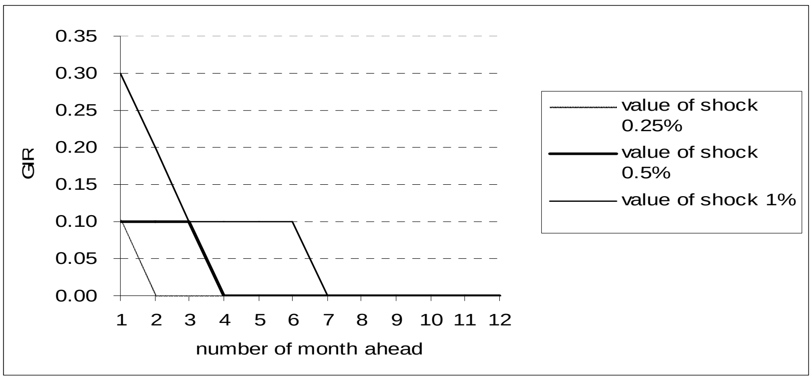

Fig. 3. The generalized impulse response of inflation in the face of interest rate shocks.

7. Conclusions

In this paper we determined the optimal monetary policy transmission horizon using the traditional reduced rank vector autoregresson model for monetary policy. Analyzing the generalized impulse response of inflation we conclude this optimal horizon is primarily dependent on the size of the shock, rather than its type.

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