Mortality models of insured population in the Slovak Republic

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Abstract

Paper presents the results of mortality modelling by age of insured persons in the Slovak republic based on data covering the period 1999-2010 submitted to the National Bank of Slovakia by insurers with exposition against this risk. Using appropriate methods of smoothing (graduation) were generated models of mortality of the insured population separately for males and females and for both gender together. Article contains a comparison of these models, as well as their comparison with analogous models, created on the basis of data obtained from the Slovak Statistical Office. At the end of this article there are the examples of quantifying differences in life insurance premiums, calculated on the basis of different mortality tables.

Keywords: *mortality rates, gender, graduation, smoothing, testing, comparison, premium*

JEL Classification: *C41, C42, C16*
AMS Classification: 65D10, 62H15

1. Introduction

On 13 December 2004 the Council of the European Union adopted The Directive 2004/113/EC implementing the principle of equal treatment between men and women in the access to and supply of goods and services with the aim to establish a framework to eliminate the discrimination on the basis of sex in the access to and supply of goods and services.

According to the Article 5 of the Directive the use of sex as a factor in the calculation of insurance premiums and benefits shall not result in differences in individuals' premiums and benefits in contracts concluded after 21 December 2007. On 4 November 2008, the National Bank of Slovakia issued Decree No. 20/2008 on submitting of insurance-mathematical and statistical data of insurance undertaking and the branch of a foreign insurance undertaking, on the basis of which it started to gather statistical data about insured people from insurance undertakings in 2009.

The data were gathered in classification according to sex, age and thirteen insurance risks. The data concerning to mortality were sent by the insurance undertakings that are authorized to perform life assurance and the data concerning other risks than mortality risk were sent by

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the insurance undertakings that use sex as one of determining factors at determination of the insurance tariffs.

Insurance sector of the European Union is currently working on implementation of Directive 2009/138/EC Solvency II. The basic purpose of this Directive is a detailed examination and valuation of risk. We consider as a paradox that anti-discrimination legislation denies these principles by removing key risk factor. Inappropriate use of anti-discrimination means competitive disadvantage for insurance companies from EU Member States. It is expected outflow of clients to insurance companies outside the EU, where the differentiation of premiums based on sex is prohibited.

2. Theoretical principles of mortality modelling

The data required to evaluate mortality involve two types of current and historical information collected by gender and age:

- the size of the population in question (the exposure to risk) E_x,
- the number of deaths occurring within that population in a given period D_x.

The exposure-to-risk in a given year refers to the total number of person-years in that population during that calendar year. For a life insurer, individuals join the insured population when they buy a policy and they leave the population when the policy lapses, is cashed in or matures or when the individual eventually dies.

In our analyse we use the data obtained from the Slovak insurance companies which collected and published mortality data for the above purposes. The published data allows calculating two basic measures:

- crude rate of mortality m_x by age x,
- graduated initial rate of mortality q_x by age x.

In practice the data generally used for calculating population mortality are:

- the number of deaths D_x over the year of individuals who were aged x last birthday,
- the number of lives E_x aged x last birthday at the middle of the year, which serves as a proxy for the exposure-to-risk.

Dividing the first of these by the second produces an estimate for the "crude rate of mortality", denoted m_x, from which q_x can be approximated. So

$$m_x = \frac{D_x}{E_x}. \tag{1}$$

This measure of mortality is different from the probability measure because the denominator for m_x only counts exposure to death for the fraction of the year for which the person was alive. By contrast the q_x denominator counts exposure to death as a whole year whether or not the person survives the entire year. It follows that a simple approximation for q_x assuming a uniform distribution of deaths over the year, is

$$q_x \approx \frac{m_x}{1+\frac{m_x}{2}}. \tag{2}$$

The actuaries of life insurance companies must use mortality measures suitable for ordinary assurance contracts, and a different mortality basis appropriate to annuitants when he determines annuity purchase prices. The construction of a new mortality table is rather more complicated. The main problem is the adjustment of the observed rates to produce smooth decrement rates which are accurate estimates of the underlying mortality. Graduation of mortality rates is useful because the raw mortality data can contain isolated features that cannot be explained rationally, as well as other sources of statistical noise.

A number of different graduation methods are available, for example the graphic method, summation and adjusted average methods, graduation by mathematical formula, graduation by reference to a standard table or spline function.

Procedure Time Series – Smoothing of the Statgraphics Centurion XVI statistical package contains ten different methods of graduation with possibility to use them twice. We have smoothed crude mortality rates by twice repeating method 5RSSH.

Methods 3RSS, 3RSSH, 5RSS, 5RSSH and 3RSR there are nonlinear smoothers developed by John Tukey (1977). These smoothers are very good at ignoring outliers, and are often applied as a first step to reduce the influence of potential outliers before a moving average is applied. Each symbol in the name of the smoother indicates an operation that is applied to the data:

"5": A moving window of size 5 is passed through the data, and the median of the 5 values in the window is kept as the smooth.

"R": The smoothed data is resmoothed by applying the previous operation to the smoothed result. This is repeated until no further change occurs.

"S": Since running medians tend to leave flat mesas and dales, these flat spots are split to make the result smoother than it would otherwise be. The main smoothing operation is then performed again.

"H": The data is smoothed using a weighted average of the form

$$S_t = 0.25 y_{t-1} + 0.5 y_t + 0.25 y_{t+1} . \quad (3)$$

3. Testing quality of graduation

Graduation tests review whether the deviations between the original data and the graduated data are randomly distributed, independent, and distributed in accordance with the assumptions inherent in the underlying model. We outline a number of tests which can be considered for the evaluation of smoothed data.

We assumed that all lives of a particular age „*x*" are subject to the same fixed probability of death q_x. This leads immediately to the binomial distribution as the natural model for the observed number of death D_x at age *x* with parameters E_x and q_x and its distribution can be approximated by a normal random variable with mean $E_x q_x$ and with variance $E_x q_x(1 - q_x)$. Deaths at successive ages are independent, and it follows that the standardized deviations

$$Z_x = \frac{D_x - E_x \dot{q}_x}{\sqrt{E_x \dot{q}_x \dot{p}_x}} \quad (4)$$

should resemble independent observations on the unit normal distribution. If we use graduated mortality rates $\dot{q}_x$ we should check, therefore, that not more than about 5 per cent of the standardized deviations exceed two in absolute size.

The existence of a number of excessively large deviations over part or the whole of the age range can be detected by comparing

$$\chi^2 = \sum_x \frac{(D_x - E_x \dot{q}_x)^2}{E_x \dot{q}_x \dot{p}_x} \quad (5)$$

with chi-square on one degree of freedom. This test examines how closely the graduation fits the original data. Graduation is suitable, if $\chi^2 < \chi^2_{1-\alpha}$.

Signs test compares the raw data to the graduated rates and, at each age, determines whether the sign of the deviation is positive or negative. The test identifies the number of positive and negative deviations and determines whether there is an imbalance between them. Under the assumption that the deviations are random and independent, the number of positive or negative signs should be binomially distributed with both signs being equally likely to occur. The number of positive (or negative) deviations in a sequence of n deviations is therefore binomial with parameters *n* and 0.5.

Runs test (Grouping of signs) looks at the number of groups of deviations of the same sign and compare this with the number that would be expected if the positive and negative signs were arranged in a truly random order. If the graduation technique is appropriate, there should be neither too few nor too many runs of successive deviations with the same sign. The number of sign changes in a sequence of n deviations is binomial with parameters $n - 1$ and 0.5.
Stevens' test for the grouping of signs is efficient in detecting the grouping of positive and negative signs. We suppose n_1 positive and n_2 negative signs. Let us imagine that our experience has g positive groups, it is possible to deduce that the mean M^+ and variance V^+ of the number of positive groups are given as

$$M^+ = \frac{n_1(n_2 + 1)}{n}, \qquad V^+ = \frac{(n_1 n_2)^2}{n^3}. \tag{6}$$

Stevens' test can be performed approximately by testing statistic

$$G = \frac{g - M^+}{\sqrt{V^+}} \tag{7}$$

comparing with the 5 per cent region of the unit normal curve, i.e. $G < -z_{1-\alpha}$.

4. Results of application above mentioned methods

In order to achieve the objectives, we used the data from the Slovak insurance companies about the number of insured persons to death and the number of deaths at age *x*, for *x* = 0, 1, ..., 100, separately for men and women in the time period 1999-2010.

These data allow us to calculate crude rate of mortality m_x according to (1) and measures of mortality q_x according to (2). Using procedure Time Series – Smoothing of the Statgraphics Centurion XVI statistical package, by twice repeating method 5RSSH for graduation we obtained graduated (smoothed) of mortality rates for men, women and for both genders together. The results of these procedures present the figures 1-3.

Number of data provided by the Slovak insurance companies is not sufficient enough for accurate estimates of the probability of death by crude mortality rates in accordance with the law of large numbers. This is especially true for high ages, where we can see great differences (roughs) of real and smoothed mortality measures for males, females and both genders groups together. Nevertheless, the tests of accuracy of graduation specified in paragraph 3 confirmed its acceptance (table 1).

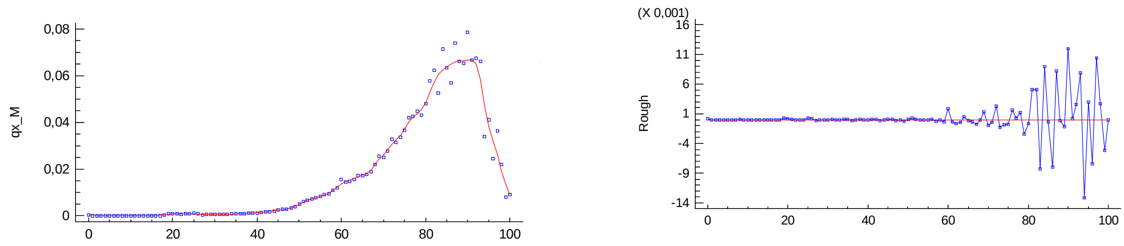

Fig. 1. Real and smoothed mortality rates and their differences for males.

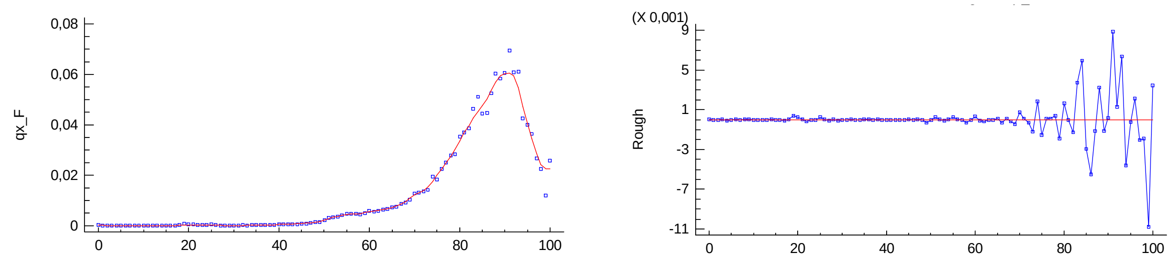

Fig. 2. Real and smoothed mortality rates and their differences for females.

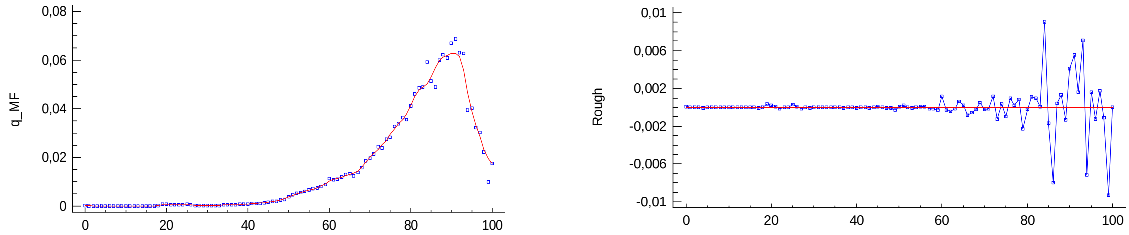

Fig. 3. Real and smoothed mortality rates and their differences for males and females together.

Gender	Z test (p-Value)	χ^2 test (p-Value)	Signs test (p-Value)	Runs test (p-Value)	Stevens' test (G)
Male	0.652	0.043	0.691	0.764	0.514
Female	0.844	0.886	0.842	0.133	1.696
Male+Female	0.973	0.598	0.691	0.484	-0.683

Table 1 Results of testing quality of smoothed data.

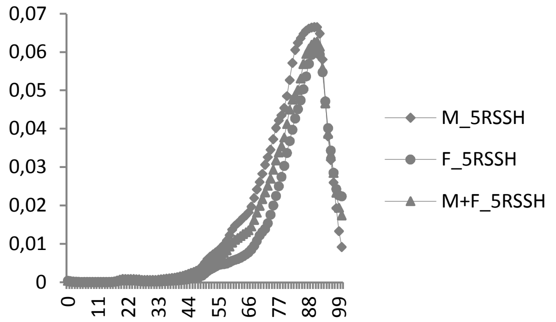

Fig. 4. Graphical comparison of graduated mortality rates by gender.

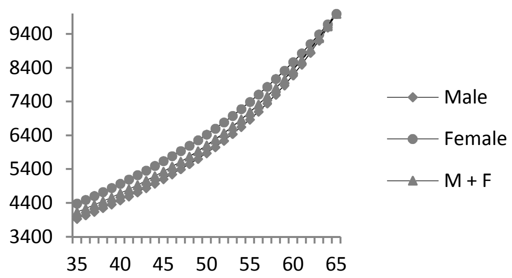

Fig. 5. Differences in single premiums by gender – pure endowment insurance.

Figure 4 shows all three mortality models on the common graph. In this picture there are clearly significant differences in mortality rates of men and women, and together of course, especially in the age range from 45 to about 90 years of age.

Gender differences in mortality rates necessarily result in differences in the single premium amount of pure endowment insurance (Fig. 5) and of term life insurance (Fig. 6). These graphs show the significant differences in insurance premiums for mention above types of insurance of the insured person under the age of 65 years at entry age 35 or more. Supposed insured sum is equal to 10 000 €.

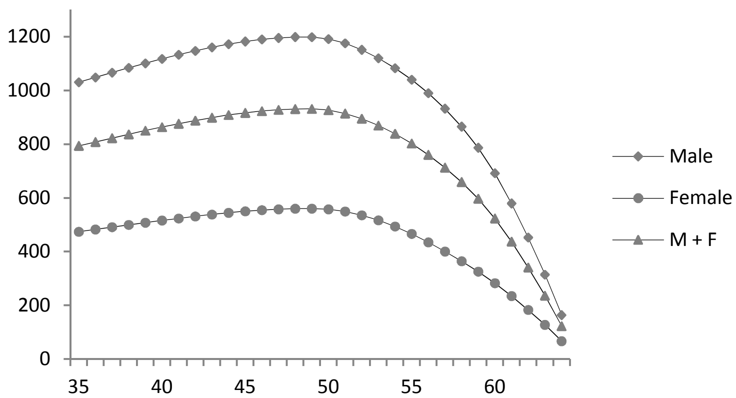

Fig. 6. Differences in single premiums by gender – term life insurance.

The following figures 7-8 show comparisons of single premiums for term life insurance and for pure endowment insurance, which is calculated using the smoothed mortality rates of the insured population and using national mortality tables 2005.

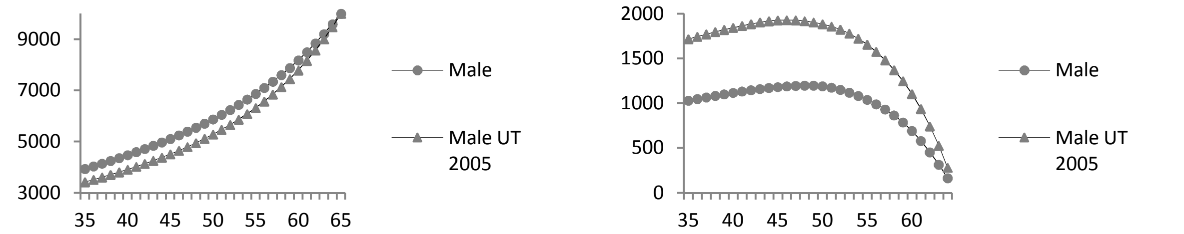

Fig. 7. Comparisons of single premiums for pure endowment and for term life insurance by different life tables for males.

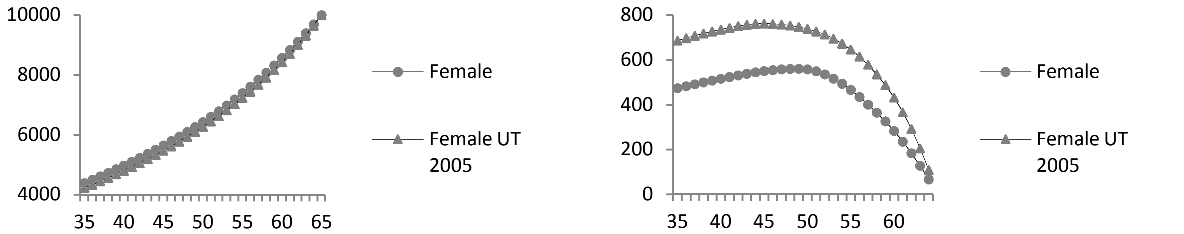

Fig. 8. Comparisons of single premiums for pure endowment and for term life insurance by different life tables for females.

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