# A β-convergence analysis of European regions. Some re-specifications of the traditional model

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**Abstract**

The aim of the paper is to investigate the economic convergence of per capita incomes across the European Union regions at a NUTS-2 level over the period 1995–2009. To achieve the purpose some selected models offered by spatial econometrics are used. The models create some re-specifications of the traditional model of β-convergence. Taking into account the spatial tendencies and dependence across the regions in the growth models allows to evaluate the convergence phenomenon on the grounds of β parameter estimated better than in the traditional approach. Then the estimate of the parameter will not be influenced by omitting the dependence. Simultaneously the importance of the neighbour connections and the spatial heterogeneity in verifying the economic convergence hypothesis will be exposed.

***Keywords:*** *β-convergence, spatial dependence, the spatial lag model, the spatial error model.*

***JEL Classification***: C52

***AMS Classification***: 62M30

## 1. Introduction

The paper presents the analysis of per capita GDP across the European Union regions at a NUTS-2 level over the period 1995–2009. The aim of the analysis is to verify the economic convergence in the area of the European countries in the investigated period.

The economic convergence hypothesis assumes equalizing over time the levels of per capita income across countries or, which is more probable, across regions within a country/group of countries. Although the hypothesis is logically correct it is not always confirmed in practice. Violation of the economic convergence may be caused by the specific conditions of the course of economic processes, e.g. during economic crisis. On the other hand, faulty inference on the problem may be influenced by imperfection of the tools of the hypothesis verification.

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To achieve the purpose of the paper some selected models offered by spatial econometrics are used. The models create some re-specifications of the traditional model of β-convergence. They are needed because the traditional model's assumptions (especially, variance homogeneity in space and independence across residuals for all regions) are often unrealistic in empirical cases. As a consequence of the fact the interpretation of the convergence parameter is frequently incorrect. Thus, it is necessary to recommend the other models as effective tools for investigation of convergence phenomenon. The earlier applications of the spatial lag model and the spatial error model for the convergence analyses of Italian NUTS-3 provinces (1951–1999) and of European NUTS-2 regions (1980–1996), in context of the random fields theory, are due to Arbia [3]. The cited work was inspiration for this investigation. On economic convergence in the context of spatial econometric analyses see also e.g.: [4], [5], [8], [9], [10].

The contents of the successive sections of the paper are as follows: in Section 2 the subject and range of the investigation are defined as well as the searching hypotheses are formulated. Section 3 briefly characterizes the data used in the investigation. Section 4 presents the methodology. In the section, methods and tools of the searching hypotheses verification are pointed out. The results of the analysis are presented in Section 5. Recapitulation contains final conclusions and indicates some omitted problems.

## 2. Subject and range of the investigation

In the analysis of the per capita GDP in the area of the European Union countries over the period 1995–2009 differentials between regional income levels across the regions established according to NUTS-2 European classification system were investigated. The essential question is whether the poor regions grow faster than the rich ones. If it is true, all regions converge to the same level of per capita income. Thus, the main research hypothesis in the investigation is based on the economic-theoretic hypothesis of convergence, which is well known in literature on the subject. The attention was paid to the so-called β-convergence.

Considering the possibilities of verification of the main hypothesis there was established the view that the models which take into account the spatial tendencies and dependence allow to validate the convergence phenomenon better than the ones which ignore them.

Apart from the traditional β-convergence model in the investigation the spatial lag model and the spatial error model were used. In addition, we tried to include into the models the spatial trend component and to establish the spatial regime.

## 3. Data

In the investigation the data for 261 regions of 27 European countries were used. The data refer to the period: 1995–2009, i.e. to 15 years. They describe the per capita GDP spatial distributions and dynamics of incomes in the European Union and come from Eurostat database (ec.europa.eu/eurostat/).

Figure 1 presents the spatial distributions of per capita GDP values (expressed by log terms) at the beginning of 1995–2009 period, i.e. in year 1995 and Figure 2 illustrates the growth rates of GDP across the regions during the period.

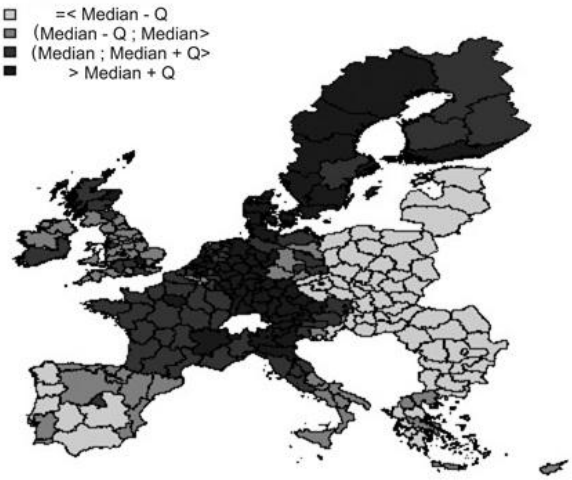


**Fig. 1.** Distribution of the per capita GDP European regions in year 1995.

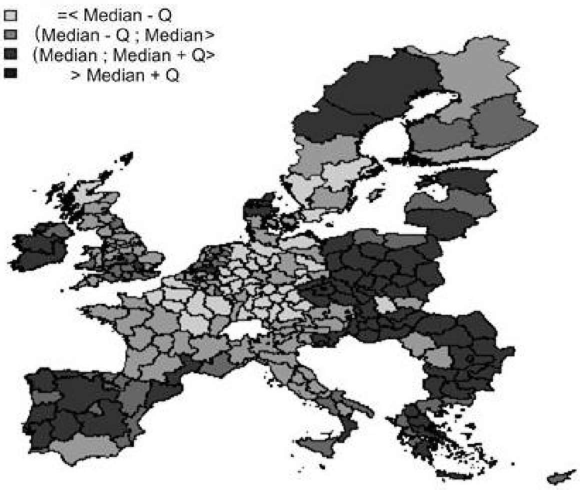


**Fig. 2.** Distribution of per capita GDP growth rates for the European regions during the period 1995–2009.

Comparing Figure 1 and 2 it can be seen that the poorer, at the beginning, regions have the faster growth rates. Thus, the economic convergence in the period considered is probable.

## 4. Methodology

The per capita GDP in the area established is treated as a realization of spatial stochastic process $Z(\mathbf{s}_i)$, where: $\mathbf{s}_i = [x_i, y_i]$ – location coordinates on the plane, $i$ = 1, 2, …, $N$ – the spatial units (regions).

To verify the β-convergence hypothesis two approaches were applied: a traditional method in which spatial connections among the regions are not considered, and the second one which takes into account the spatial connections.

In traditional approach the following model is used:

$$\ln\left[\frac{Z(\mathbf{s}_i)_T}{Z(\mathbf{s}_i)_0}\right] = \alpha + \beta \ln[Z(\mathbf{s}_i)_0] + \varepsilon(\mathbf{s}_i), \tag{1}$$

where: $Z(\mathbf{s}_i)_T$, $Z(\mathbf{s}_i)_0$ – the per capita GDP in region $i$ at time $t$, $t = T$ (last year of the period considered) and $t$ = 0 (the base period), respectively, $\varepsilon(\mathbf{s}_i)$ – the spatial white noise, i.e. $\varepsilon(\mathbf{s}_i) \sim N(\mathbf{0}, \sigma^2\mathbf{I})$.

In model (1) it is assumed that the component $\varepsilon(\mathbf{s}_i)$ is normally distributed (with zero mean and constant variance) independently of $\ln[Z(\mathbf{s}_i)_0]$. Furthermore, one assumes that $\{\varepsilon(\mathbf{s}_i)\}$, $i$=1,2, …, $n$ are independent across the locations $i$.

The second approach uses the models as follows:

$$\ln\left[\frac{Z(\mathbf{s}_i)_T}{Z(\mathbf{s}_i)_0}\right] = \alpha + \beta \ln[Z(\mathbf{s}_i)_0] + \rho \mathbf{W} \ln\left[\frac{Z(\mathbf{s}_i)_T}{Z(\mathbf{s}_i)_0}\right] + \varepsilon(\mathbf{s}_i), \tag{2}$$

$$\ln\left[\frac{Z(\mathbf{s}_i)_T}{Z(\mathbf{s}_i)_0}\right] = \alpha + \beta \ln[Z(\mathbf{s}_i)_0] + \eta(\mathbf{s}_i), \quad \eta(\mathbf{s}_\mathbf{i}) = \lambda \mathbf{W} \eta(\mathbf{s}_i) + \varepsilon(\mathbf{s}_i), \tag{3}$$

where: $Z(\mathbf{s}_i)_T$, $Z(\mathbf{s}_i)_0$, $\varepsilon(\mathbf{s}_i)$ – as above, **W** – the spatial connectivity matrix.

Models of the forms (2)–(3) represent the so-called spatial lag model and spatial error model, respectively, which are often used in the spatial econometric literature (methodological aspects are reviewed e.g. in [1] and [2]).

If the spatial trend is also identified, the component $P(\mathbf{s}_i)$ should be included into the models. The trend component may be described with the polynomial function of degree $p$ in the following general form:

$$P(\mathbf{s}_i) = \sum_{k=0}^{p} \sum_{m=0}^{p} \theta_{km} x_i^k y_i^m \,, \qquad (4)$$

where: $k + m \leq p$.

Economic convergence is said to be confirmed by the data if the estimate of β coefficients in models (1)–(3) are negative and statistically significant.

Taking into consideration the spatial tendencies and dependence across the regions in the growth models allows us to evaluate the convergence phenomena on the grounds of β parameter estimated better than in the traditional approach. Then the estimate of the parameter will not be influenced by omitting the dependence.

The parameter β may be used to establish the so-called *half-life time* defined as the time that is necessary for $\ln[Z(\mathbf{s}_i)_t]$ to be half way between the initial value $\ln[Z(\mathbf{s}_i)_0]$ and the value $\ln[Z(\mathbf{s}_i)_T]$ (see, [3], pp.11-13). It is expressed as: $t_{half-life} = \frac{\ln(2)}{b} = 0.69b^{-1}$, where: $b = -\frac{\ln(1+\beta)}{T}$ – *speed of convergence*.

In order to evaluate the quality of the empirical models the following tools were used: the Moran test (Moran's *I*) for spatial independence, the Lagrange Multiplier tests ($LM_{lag}$, $LM_{err}$) and their robust versions ($RLM_{lag}$, $RLM_{err}$) as spatial dependence diagnostics, the Likelihood Ratio test (LR) for testing the significance of the spatial dependence, the Breusch-Pagan heteroskedasticity test.

## 5. Results

In this section the main results of the performed investigation are presented. They refer to validity of selected econometric models as the tools of β-convergence hypothesis verification.

The successive tables presented below contain the information on usefulness of three conceptions: the linear regression model, the spatial lag model and the spatial error model. The spatial trend components had been included into the models but they proved to be insignificant. Thus, the presentation of them was omitted.

To investigate β-convergence phenomenon in traditional approach the model (1) was used. Table 1 contains the results of estimation and verification of this model. The estimate of the β coefficient is negative and statistically significant. Unfortunately, the Breusch-Pagan statistic is significant, leading to the rejecting the homoskedasticity assumption of the model. In addition, on the basis of the Moran's *I* test it is necessary to state that the hypothesis of independence of the traditional model residuals should be rejected. This conclusion has been confirmed by the graphical presentation of the residuals distribution in Figure 3.

| **Parameters** | **Estimates of parameters** | **Standard errors** | **Statistics *t*** | **Pr (>\|t\|)** |
|---|---|---|---|---|
| α | 3.8739 | 0.1139 | 34.0000 | 0.0000 |
| β | -0.3535 | 0.0122 | -29.0500 | 0.0000 |
| Speed of convergence: 0.0291 | | | | |
| Half-life: 23.8477 | | | | |
| Adjusted R-squared: 0.7642 | | | | |
| F-statistic: 843.7, p-value: 0.0000 | | | | |
| Breusch-Pagan heteroskedasticity test: 11.0877, p-value: 0.0009 | | | | |
| Autocorrelation of residuals: Moran test: 5.4531, p-value: 0.0000 | | | | |

**Table 1** The results of the model (1) estimation and verification.

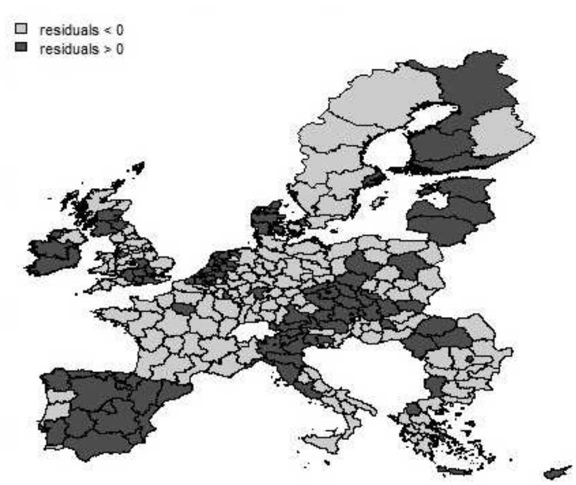


**Fig. 3.** Spatial distribution of the residuals of model (1).

The residuals are not distributed randomly but they are dependent on their geographical locations. The map displays a marked geographical regularity. There are subspaces in which the residuals are positive and the others characterized by negative residuals. This means that in the regions located in some portions of the geographical space per capita incomes are underestimated (positive residuals) by the model, while in the other regions the incomes are overestimated (negative residuals) by the β-convergence regression. Reassuming, as the model considered does not satisfy the theoretical assumptions, other models should be proposed.

As the Moran test does not admit an explicit alternative hypothesis opposed to the null, the Lagrange Multiplier tests (LM) were used. Table 2 contains the results of testing the hypothesis of spatial independence using the tests that consider the spatial lag model and the spatial error model as alternatives ($LM_{lag}$ and $LM_{err}$, respectively). Moreover, it reports the results of using the robust tests ($RLM_{lag}$, in which $H_0$: $\rho = 0$ under the assumption that $\lambda \neq 0$ and $RLM_{err}$, where $H_0$: $\lambda = 0$ under the assumption that $\rho \neq 0$). All LM tests are significant and the following re-specifications are justified.

| **Tests** | **$LM_{lag}$** | **$LM_{err}$** | **$RLM_{lag}$** | **$RLM_{err}$** |
|---|---|---|---|---|
| **Values of test** | 26.3535 | 29.7552 | 4.0923 | 7.4940 |
| **p-value** | 0.0000 | 0.0000 | 0.04308 | 0.00619 |

**Table 2** Spatial dependence tests for the OLS residuals of the β-convergence model.

| **Parameters** | **Estimates of parameters** | **Standard errors** | **Statistics t** | **Pr (>\|t\|)** |
|---|---|---|---|---|
| α | 2.8602 | 0.2138 | 13.3760 | 0.0000 |
| β | -0.2618 | 0.0202 | -12.9730 | 0.0000 |
| Speed of convergence: 0.0202; Half-life: 34.2531 | | | | |
| ρ = 0.2860 | | | | |
| test LR: 27.1540, p-value: 0.0000 | | | | |
| Wald statistic: 30.08, p-value: 0.0000 | | | | |
| AIC: -191.86 (AIC for lm: -166.71) | | | | |
| Breusch-Pagan heteroskedasticity test: 13.1741 , p-value: 0.0003 | | | | |
| Autocorrelation of residuals: Moran test: 2.038, p-value: 0.04155 | | | | |

**Table 3** The results of the model (2) estimation and verification.

In the economic β-convergence analysis of the European NUTS-2 regions the models (2) and (3) were used as well. The models take into account the spatial connections among the regions. The results of estimation and verification of the model (2) are displayed in Table 3. Next, Table 4 contains analogical results for the model (3). Diagnostics for the models considered suggest that the spatial error model is the best of them.

| **Parameters** | **Estimates of parameters** | **Standard errors** | **Statistics t** | **Pr (>\|t\|)** |
|---|---|---|---|---|
| α | 3.6383 | 0.1702 | 21.3750 | 0.0000 |
| β | -0.3269 | 0.0181 | -18.0130 | 0.0000 |

Speed of convergence: 0.0264; Half-life: 26.2654

λ = 0.4548

test LR: 30.623, p-value: 0.0000

Wald statistic: 47.131, p-value: 0.0000

AIC: -195.33 (AIC for lm: -166.71)

Breusch-Pagan heteroskedasticity test: 4.0292 , p-value: 0.045

Autocorrelation of residuals: Moran test: -0.0946, p-value: 0.9247

**Table 4** The results of the model (3) estimation and verification.

## 6. Recapitulation

The settlements of the paper show that including into the economic convergence model the spatial dependence components (spatially lagged dependent variable or the spatial autoregressive error term) is justified and very important for the analyses of per capita incomes in regions and the incomes changes over time.

However, the empirical models received are not entirely satisfactory, because of heteroskedasticity of variance. The spatial heteroskedasticity can be caused by omitting the factor responsible for systematic spatial variability. The spatial trend component in the models has not solved the problem. In further investigation the spatial regimes will be searched for (comp. Arbia [3] and his considerations on spatial changeability of the model parameters).

In the paper simultaneous investigation of temporal and spatial heterogeneity or autocorrelation and especially of space-time dynamics in β-convergence models was omitted. On the questions see e.g. [6], [7].

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