# Analysis of cartel equilibrium model and guarantee the conditions of economic competition

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**Abstract**

A cartel is generally perceived as a specific market form of oligopoly where cartel subjects accede to a discreet agreement between formally legally independent economic subjects which together enter into a contract with an aim of reaching a more favorable position on relevant market and thus eliminate the mechanism of competition. Cartels which, based on the agreed market strategies, may follow common price strategy, set their production quotes or divide the market are forbidden in the European Union countries as well as in many countries outside the EU. This functional scheme of cartel behavior is thus fundamentally incompatible with generally supported terms of competition protection on the markets of developed economies. In the developed economies of the European Union and the world there are established governmental institutions to control and guarantee the conditions of economic competition.

In this article we will present the mathematical formalization of the model of equilibrium price and supply of the cartel agreement participants and point out the social inefficiency of such decision-making scheme. In more detail we will discuss the analysis of the price cartel considering various costs of the producers.

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## 1. Introduction

A cartel is a specific case of oligopoly with an unspecified number of buyers but only a small number of sellers. The upper limit of the number of the sellers in the market structure for it to be defined as oligopoly is not explicitly defined. The key issue is not the number of the sellers but the way they communicate with each other, how they react to their individual intentions and how they jointly address the conditions and attributes of the oligopoly market equilibrium, that is how they resolve the question of market price of their products, how they set the total supply of the sector and how they agree on the individual contribution of the oligopoly subjects to the creation of total oligopoly supply on the relevant market.

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It should be noted that each action of a particular firm in an oligopoly affects the behavior of other firms on the market. Price lowering of one firm will likely decrease a market share of other firms on the production of a sector. In other words, responses of the competitors in oligopoly may have a significant effect on a result of managerial decision making on an oligopoly market. It is therefore clear that a decision making about optimal price and supply in an oligopoly is far more complicated than in other market structures.

Let's mention two other characteristics which describe market structures – product differentiation and conditions for entering and exiting a sector. A product sold in the oligopoly may be either homogenous or differentiated. If a product is homogenous we say that a market represents *pure oligopoly*. Unlike pure oligopoly, in case of homogenous product we speak of a *differentiated oligopoly*.

In [5] we can see that the most typical features of an oligopoly that characterize this market structure and differentiate it from other types of market structures are a necessity of communication between oligopoly subjects, identification of interdependencies and a search for generally acceptable solutions. An action of one seller may affect another seller and thus make the other seller react in a way which affects the first seller in return. Firms in an oligopoly will seek mutual agreement based on various assumptions and schemes of alignment of interests which will be determined by specific characteristics of a sector.
Methodological problems of formulating oligopoly models rise from the great diversity of ways in which firms can interact and conclude agreements on the distribution, market shares and market prices. Simply said, there is no general model of oligopoly.

However there are models which analyze oligopoly decision making based on certain assumptions about interactions between firms. In this paper we examine a general scheme of oligopoly equilibrium model on which we present specific aspects of mutual relations between oligopoly subjects in a process of setting an equilibrium price and supply of oligopoly.

In the paper we present the results of microeconomic analysis of a cartel equilibrium model and show how a method of setting an optimal price and supply of a cartel limits the laws of competition.

## 2. General equilibrium model of supply and price on the oligopoly market

To begin our analysis we examine an oligopoly market structure where on the supply side there are $n$ producers who conclude a cartel agreement on market sharing and price and supply optimization. We also assume that the number of buyers forming demand is unlimited.

Therefore any communication let alone common coordinated actions of buyers are ruled out. Behavior of the buyers in the market considering a supply of a homogenous product of $Q$ units at an acceptable price $P$ is described by a price-demand function of an average buyer in a relevant market as follows:

$$p(q) \tag{1}$$

where: $q \in R$ – volume of supply/demand, $p \in R$ – product price, $p(q)$ – real price-demand function, $p(q)$: R→R, while price-demand function indicates that an average buyer is willing to buy $q$ units of a product at a unit price $p$. Every producer in oligopoly supplies $q_i$ $i$=1, 2,..., $n$ units of homogenous production and a total supply in the sector is $q = \sum_{i=1}^{n} q_i$ .

At first let's examine separately the behavior of each of the producers in a sector having an interest to conclude a cartel agreement and let's interest in an analytical presentation of conditions which need to be fulfilled for these producers to reach a maximum profit without having to use this agreement. Fendekova and Fendek [2] shows that each producer in a sector has his own individual technological production conditions defined by the cost function $tc_i(q_i)$, which must guarantee such costs for amount of production $q_i$, that represent minimal production costs:

$$c_i(q_i) = vc_i(q_i) + f_i \quad \text{for } i = 1, \dots, n \tag{2}$$

where: $q_i$ – volume of supply of an $i$ th oligopoly producer, $q_i \in$ R, $c_i(q)$ – real function of total costs of an $i$ th oligopoly producer, $c_i$: R→R, $vc_i(Q)$ – real function of variable costs of an $i$ th oligopoly producer, $vc_i$: R→R, $f_i$ – fixed costs of an $i$ th oligopoly producer.

While a price of production $q_i$ of an $i$ th producer is also influenced by volume of supply of other producers $q_j$ for $j$=1, ..., $n$, $j \neq i$ therefore an acceptable price for the market for which an $i$ th producer sells his production $q_i$ is given by:

$$p_i = p(q) = p\left(\sum_{i=1}^{n} q_i\right) = p(q_1 + q_2 + \cdots + q_n) \tag{3}$$

and $p_i = p = const, \qquad i = 1,2, \dots, n$

A price of a homogenous product is therefore constant for all the producers in an oligopoly. We can see that a major complication which each producer in an oligopoly must deal with while setting a price of his production is that his production price is not only

a function of his own supply $q_i$ but also of supplies $q_j$ for $j = 1, 2, ..., n, j \neq i$ of the other oligopoly producers while the price in the end is the same for all oligopoly producers (but in a special class of models of a price leader with a residual demand). So the revenue function of the *i* th producer in an oligopoly is:

$$r_i(q_i) = p\left(\sum_{i=1}^{n} q_i\right) q_i = p(q_1 + q_2 + \cdots + q_n) q_i \tag{4}$$

and the profit function of the *i* th producer in an oligopoly is

$$\pi_i(q_i) = r_i(q_i) - c_i(q_i) = p\left(\sum_{i=1}^{n} q_i\right) q_i - c_i(q_i). \tag{5}$$

Note that a revenue function of the *i* th producer in an oligopoly de facto is a function of one variable $q_i$ because in a price-demand function

$$p_i = p(q) = p\left(\sum_{i=1}^{n} q_i\right) = p(q_1 + q_2 + \cdots + q_n)$$

only his own supply $q_i$ is a variable while the supplies $q_j$ for $j = 1, 2,..., n, j \neq i$ of the other oligopoly producers are parameters of this function. A problem of solving the optimality conditions of the *i* th producer then represents a way to find such an optimal supply $q_i^*$ of the th producer given a production price $p^*$ for which the producer reaches maximum profit

$$\pi_i(q_i) = r_i(q_i) - c_i(q_i) \rightarrow max. \tag{6}$$

Problem (6) represents unconstrained optimization problem and assuming that the functions used in a model are smooth i.e. continuous and differentiable, a necessary condition for an extremum of the profit function (5) of the *i* th producer in an oligopoly is:

$$\frac{\partial \pi_i(q_i)}{\partial q_i} = \frac{\partial r_i(q_i) - \partial c_i(q_i)}{\partial q_i} = \frac{\partial p(\sum_{i=1}^{n} q_i) q_i}{\partial q_i} - \frac{\partial c_i(q_i)}{\partial q_i} = 0 \quad i = 1, \ldots, n \tag{7}$$

and a sufficient condition for a maximum, i.e. strict concavity, of a profit function for a stationary point $q_i^0$ gotten from a solution of an equation for necessary condition for an extremum (7) is:

$$\frac{\partial^2 \pi_i(q_i)}{\partial {q_i}^2}_{[q_i = q_i^0]} < 0. \tag{8}$$

It can be shown that a way to solve an equilibrium problem (6) of a firm in an oligopoly is determined by whether the conditions in an oligopoly are competitive for a producer, meaning a producer faces a risk of other producer setting a different, seemingly lower, price of their production, which is a standard hypothesis for an oligopoly, or a producer is economically strong enough to potentially gain a position in an oligopoly and dictate the conditions or in extreme case take a position resembling a monopoly. Pepall, Richards, Norman [5] shows, that in both cases a firm in a dominant position in an oligopoly sets the price individually in accordance with its own visions based on its own technological production conditions.

In the first case of competitive relations in an oligopoly the *i* th individual producer sets his optimal supply $q_i^{CO}$ depending on common market price of the production $P_{CO}$ based on a solution of a following equation of optimality condition of a firm in a competitive environment for maximization of its profit

$$q_i^{CO}: p_{CO} = p\left(\sum_{i=1}^{n} q_i\right) = \frac{\partial c_i(q_i)}{\partial q_i}. \tag{9}$$

In the second case of a noncompetitive environment in an oligopoly the *i* th individual producer, having a dominant position in an oligopoly, sets his optimal supply $q_i^{OL}$ depending on a market price of the production $P_{OL}$ that only he dictates, based on a solution of a following equation of optimality condition of a dominant firm in an oligopoly environment for maximization of its profit

$$q_i^{OL}: p_{OL} = p\left(\sum_{i=1}^{n} q_i\right) = \frac{e(q_i)}{1 + e(q_i)} \frac{\partial c_i(q_i)}{\partial q_i}, \tag{10}$$

where $e(q_i) : R \rightarrow R$ is a real function of an own price elasticity of a demand for homogenous product assuming that the demand is elastic and a value of an own price elasticity function satisfies $(q_i) > 1$, resulting into

$$\frac{e(q_i)}{1 + e(q_i)} > 1. \tag{11}$$

Needless to say, it is not simple to clearly decide which of the two general principles should be applied while quantifying oligopoly equilibrium of particular cases of market structures. After all, (9) and (10) formalize only the general principles of problem solution. A particular scheme of an oligopoly structure then determines particular methodology to quantify oligopoly equilibrium. Moreover, using the second scheme implicates other

questions, mainly a participation of other subjects on a process of market sharing. Obviously many other questions such as this one may be formulated.

It is obvious that, assuming an elastic demand, a price dictated by a producer in a dominant position without a risk of a competitors' pressure to lower the price is always higher than in a case of a competitive environment in an oligopoly. In general

$$p^{OL} > p^{CO}. \tag{12}$$

Solving the generally formulated problems based on (9) and (10) allows quantifying oligopoly equilibrium. Particular process of solution depends on assumptions which are used in strategic interactions of the firms in oligopoly. Three scenarios are possible:

(a) Each firm in oligopoly decides about its production strategy under an assumption that it can guess the production strategies of other firms. Their choice of strategies must in the end converge to a state acceptable for all the subjects in an oligopoly. Based on this strategic interaction their positioning is a convenient consensus for all the firms.

(b) Position of one firm in an oligopoly is strong enough in a sector and relevant market for other firms to respect its hegemony on the market and to form their production strategies based on information about stronger firm decisions. This decision making scheme of production strategies of an oligopoly is described in solution of an oligopoly equilibrium problem as a relation of a leader and followers.

(c) The above described groups of an oligopoly equilibrium models assumed that the subjects in an oligopoly are in a competitive relation and based on a competition they struggle for the best market position and that a final state is an outcome of their strategic interactions on the market.

There is one other specific situation which, however, represents a violation of competition principles. Firms in an oligopoly may close an agreement on collusion – an agreement on a division of the market or on a price of production with an objective of common oligopoly profit maximization. Part of this agreement is a scheme of the profit division between the oligopoly subjects.

Optimality conditions of equilibrium price and supply problem (7) in the end represent a system of *n* in general nonlinear equations in *n* variables. The solution of this system represents so called *pareto optimal* division of a market between the producers where each of the producers may increase their market share only when some of the remaining producers worsens his position.

## 3. Model of supply and price equilibrium in a cartel

Let's now study a situation in a sector where we assume the individual producers agree on a common course of action while setting a volume of their production, market shares and market price. So the producers will de facto act as a monopoly even though formally they remain independent firms.

If the producers ignored a behavior of the other producers while making their decisions, every firm would independently solved its own profit maximization problem in a form (6) based on optimality conditions (7).

If, however, the firms won't proceed independently but agree on common approach to production and sales, by which they violate the conditions of competition.

So the firms agree on a common course of action. They will seek such a solution of equilibrium on a market that would maximize their common profit and adjust their individual production strategies to this solution. They will therefore seek optimal volumes of supply of their production based considering common profit function maximization in a form:

$$\pi(q) = \pi\left(\sum_{i=1}^{n} q_i\right) = \sum_{i=1}^{n} \pi_i(q_i) = \sum_{i=1}^{n} r_i(q_i) - \sum_{i=1}^{n} c_i(q_i). \tag{13}$$

After expressing a price of cartel production using price-demand function (3) in *n* variables we transform a common profit function maximization problem (13) of cartel subjects to a problem in a form:

$$\pi(q_1, q_2, \dots, q_n) = \sum_{i=1}^{n} p(q_1, q_2, \dots, q_n) q_i - \sum_{i=1}^{n} c_i(q_i) \to max. \tag{14}$$

Optimization problem (14) represents an unconstrained common profit function maximization problem in *n* variables and is realized in a stationary point of a concave function of a common cartel profit satisfying necessary optimality conditions in a form:

$$\frac{\partial \pi(q_1, q_2, \dots, q_n)}{\partial q_i} = \frac{\partial\left(\sum_{i=1}^{n} p(q_1, q_2, \dots, q_n) q_i - \sum_{i=1}^{n} c_i(q_i)\right)}{\partial q_i} \quad i = 1,2, \dots, n. \tag{15}$$

A degree of difficulty of a solution of necessary optimality conditions defined by a set of nonlinear equations (15) is, naturally, determined by a degree of difficulty of a price-demand function of a sector and cost functions of particular producers – cartel subjects.

## 4. Conclusion

An oligopoly represents a market structure where a limited number of producers operate on a market of a sector. Firms in an oligopoly must respect an existence of their competitors and seek mutual agreement based on different assumptions and schemes of aligning the interests which will be determined by specific characteristics of a sector. In this competition scheme an existence of an oligopoly in accordance with the rules of economic competition is socially effective and a presence of the competitive relations on a producers' side is naturally effective as well. A different situation occurs when the firms in an oligopoly which formally declare their independency and the existence of competitive relations in a sector, close a secret agreement on joint process of fixing a volume of supply and price of a product in a sector, which is clearly against the principles of a competition protection. In this case we speak of a cartel. An aim of this practice is an effort to reach an extra profit for cartel subjects.

If the producers in a sector close the agreements based on cartel principles, they significantly limit a quality of a competition and it is an obligation of a state to regulate or eliminate this condition by legislation. In this paper we dealt with methodological tools of microeconomic analysis for fixing an optimal supply and price of a cartel and showed the ineffectiveness of such market structure.

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