Application of the original price index formula in the CPI commodity substitution bias measurement

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Abstract

In this paper we propose the application of the original, superlative price index formula for the measurement of commodity substitution bias associated with Consumer Price Index (CPI). In our simulation study we compare CPI bias values calculated by using the original price index formula with those calculated on the basis of some known, superlative price indices.

Keywords*: CPI, COLI, superlative index, Laspeyres index, Fisher index*

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1. Introduction

The Consumer Price Index (CPI) is used as a basic measure of inflation. The index approximates changes of costs of household consumption that provide the constant utility (COLI, Cost of Living Index). In practice, we use the Laspeyres price index in the CPI measurement (White [18], Clements and Izan [6]). The Lapeyres formula does not take into account changes in the structure of consumption, which are consequences of price changes in the given time interval. It means the Laspeyres index can be biased due to the commodity substitution. Many economists consider the superlative indices (like the Fisher index or the Törnqvist index) to be the best approximation of COLI. Thus, the difference between the Laspeyres index and the superlative index should approximate the value of the commodity substitution bias. In this paper we propose the application of the original, superlative price index formula (Białek [2]) in measuring the commodity substitution bias associated with Consumer Price Index (CPI). In our simulation study we compare CPI biases calculated by using the original price index formula with those calculated on the basis of some known, superlative price indices. It should be emphasized, that we do not consider other sources of CPI bias, presented in the paper by White [18].

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2. Superlative price indices in CPI bias measurement

Any discussion on consumer price index biases must first address the important issue of the target measure with respect to which the bias is measured. The final report of the Boskin Commission begins with the recommendation that “the Bureau of Labor Statistics (BLS) should establish a cost of living index (COLI) as its objective in measuring consumer prices” (Boskin et al. [5], page 2). Further discussions on the theory of the COLI can be found in the following papers: Diewert [10], Pollak [16]). Let $E(P,\overline{u}) = \min_{Q}\{P^T Q | U(Q) \geq \overline{u})$ be the expenditure function of a representative consumer which is dual to the utility function $U(Q)$. In other words it is the minimum expenditure necessary to achieve a reference level of utility $\overline{u}$ at vector of prices P. Then the Konüs cost of living price index is defined as:

$$I_K = \frac{E(P^t,\overline{u})}{E(P^s,\overline{u})}, \tag{1}$$

where t denotes the current period, s denotes the base period and, in general, the vector of N considered prices at any moment τ is given by $P^\tau = [p_1^\tau, p_2^\tau, ..., p_N^\tau]^T$. I_K is a true cost of living index in the commodity Q changes as the vector of prices facing the consumer changes. The CPI, in contrast, measures the change in the cost of purchasing a fixed basket of goods at a fixed sample of outlets over a time interval, *i.e.* $Q^s = [q_1^s, q_2^s, ..., q_N^s]^T = Q^t$. The CPI is a Laspeyres-type index defined by

$$I_{La} = \frac{\sum_{i=1}^{N} q_i^s p_i^t}{\sum_{i=1}^{N} q_i^s p_i^s}, \tag{2}$$

so we assume here the constant consumption vector on the level from the base period. It can be shown (Diewert [11]) that under the assumption that the consumption vector Q^t solves the base period t expenditure minimization problem, that:

$$I_K = \frac{E(P^t, U(Q^s))}{E(P^s, U(Q^s))} \leq I_{La}, \tag{3}$$

so $I_{La} - I_K$ is the extent of the commodity substitution bias, where I_K plays the role of reference benchmark. In the so called economic price index approach many authors use superlative price indices to approximate the I_K index (White [18]).

First we define a price index I to be *exact* for a linearly homogeneous aggregator function f (here a utility function), which has a dual unit cost function $f(P)$ and it holds that

$$I = \frac{f(P^t)}{f(P^s)}. \tag{4}$$

In other words, an *exact* price index is one whose functional form is such that it is *exactly* equal to the ratio of cost functions for some underlying functional form representing preferences. The Fisher price index I_F (defined below as a geometric mean of the Laspeyres and Paasche indices) is exact for the linearly homogeneous quadratic aggregator function $f(x) = (x^T Ax)^{0,5}$, where A is a symmetric matrix of constants (Diewert [9]). The quadratic function above is an example of a *flexible functional form* (i.e. a function that provides a second order approximation to an arbitrary twice continuously differentiable function). Since I_F is exact for a flexible functional form, it is said to be a *superlative* index number (Diewert [9]). In the papers of Afriat [1] and Samuelson-Swamy [17] we can meet other examples of exact index numbers and also superlative index numbers (Diewert [9]). Under all above remarks, an estimate of the commodity substitution bias B_{csub} can be given by (White [18]):

$$B_{csub} \approx I_{La} - I_F. \tag{5}$$

In general, we can use any other superlative index $I_{\sup}$ to calculate the above-mentioned CPI bias, namely (Hałka i Leszczyńska [13]):

$$B_{csub} \approx I_{La} - I_{\sup}. \tag{6}$$

In this paper we compare CPI biases calculated by using some known, superlative price indices. In particular, we use the Fisher price index, the Törnqvist price index (I_T), the Walsh price index (I_W) and some original price index formula, defined in the next part of the paper (I_B). These indices are as follows:

$$I_F = \sqrt{I_{La} I_{Pa}}, \tag{7}$$

$$I_W = \frac{\sum_{i=1}^{N} p_i^t \cdot \sqrt{q_i^s q_i^t}}{\sum_{i=1}^{N} p_i^s \cdot \sqrt{q_i^s q_i^t}}, \tag{8}$$

$$I_T = \prod_{i=1}^{N} \left(\frac{p_i^t}{p_i^s}\right)^{\overline{w}_i}, \tag{9}$$

where

$$w_i^s = \frac{p_i^s q_i^s}{\sum_{k=1}^{N} p_k^s q_k^s}, \quad w_i^t = \frac{p_i^t q_i^t}{\sum_{k=1}^{N} p_k^t q_k^t}, \quad \overline{w}_i = \frac{1}{2}(w_i^s + w_i^t). \tag{10}$$

3. The original, superlative price index formula

In the paper of Białek [2] we propose the following price index

$$I_B = \sqrt{I_L \cdot I_U}. \tag{11}$$

where the *lower* (I_L) and *upper index* (I_U) we define as follows:

$$I_L = \frac{\sum_{i=1}^{N} \min(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^{N} \max(q_i^s, q_i^t) p_i^s}, \tag{12}$$

$$I_U = \frac{\sum_{i=1}^{N} \max(q_i^s, q_i^t) p_i^t}{\sum_{i=1}^{N} \min(q_i^s, q_i^t) p_i^s}. \tag{13}$$

In the above-mentioned paper it is proved that the index I_B^P satisfies *price dimensionality, commensurability, identity, the mean value test, the time reversal test* and *linear homogeneity* (Białek [3]). Moreover, there are some interesting relations between this index and other formulas. For example, in the paper by Białek [4] it is also proved that:

$$\sqrt{\frac{I_L}{I_U}} \le \frac{I_F}{I_B} \le \sqrt{\frac{I_U}{I_L}}, \tag{14}$$

that leads to the following conclusion:

$$\forall i \in \{1,2,...,N\} \;\; q_i^s \approx q_i^t \;\; \Rightarrow \; I_L \approx I_U \; \Rightarrow \; I_F \approx I_B \, . \tag{15}$$

In the paper of von der Lippe [20] it is proved that the Marshall-Edgeworth price index I_{ME} can be written as a weighted arithmetic mean of I_L and I_U, namely

$$I_{ME} = \frac{\sum_{i=1}^{N} p_i^s \max(q_i^s, q_i^t)}{\sum_{i=1}^{N} p_i^s \max(q_i^s, q_i^t) + \sum_{i=1}^{N} p_i^s \min(q_i^s, q_i^t)} I_L + \frac{\sum_{i=1}^{N} p_i^s \min(q_i^s, q_i^t)}{\sum_{i=1}^{N} p_i^s \max(q_i^s, q_i^t) + \sum_{i=1}^{N} p_i^s \min(q_i^s, q_i^t)} I_U \cdot \tag{16}$$

In fact, we can make a much more general observation – it can be proved (Białek [4]) that each of above-mentioned indices (Fisher, Laspeyres, Paasche, Marhall-Edgeworth, Walsh formulas) have values between the *lower* and *upper index.* Thus, the formula I_B, as a geometric mean of I_L and I_U, seems to be well constructed. We also show that the I_B formula is a superlative price index. For this purpose let us use the following theorem (the original version of the theorem can be found in papers: Afriat [1] or Pollack [16]).

Theorem 1

Let $Q^1, Q^2 \geq 0_N$ and let us suppose that $P^s > 0_N$ is a solution to $\max_x \{f(x) : Q^1 \circ x \leq Q^1 \circ P^s, x \geq 0_N\}$, $P^t > 0_N$ is a solution to $\max_x \{f(x) : Q^2 \circ x \leq Q^2 \circ P^t, x \geq 0_N\}$, where "$\circ$" denotes a inner product of two vectors, O_N denotes an $N-$dimensional vector of zeroes, $f(x) = (x^T Ax)^{0,5}$ for a symmetric matrix of constants A and the maximization takes place over a region where $f(x)$ is concave and positive. Then:

$$\frac{f(P^t)}{f(P^s)} = [\frac{(Q^1 \circ P^t)(Q^2 \circ P^t)}{(Q^1 \circ P^s)(Q^2 \circ P^s)}]^{\frac{1}{2}} \, . \tag{17}$$

For a proof that the functional form $f(x) = (x^T Ax)^{0,5}$ is *flexible* see Diewert [8]. Taking $Q^1 = Q^s$ and $Q^2 = Q^t$ we reach the above-mentioned conclusion, that the Fisher price index is superlative. In our case, if we take $Q^1 = [\min(q_1^s, q_1^t), \min(q_2^s, q_2^t), ..., \min(q_N^s, q_N^t)]^T$ and $Q^2 = [\max(q_1^s, q_1^t), \max(q_2^s, q_2^t), ..., \max(q_N^s, q_N^t)]^T$ in the Theorem 1, we obtain

$$I_B = \frac{f(P^t)}{f(P^s)}, \tag{18}$$

and that means that index I_B is also a superlative price index.

4. Simulation study

Let us take into consideration a group of $N = 50$ commodities.

Characteristics	$I_{La} - I_F$	$I_{La} - I_T$	$I_{La} - I_W$	$I_{La} - I_B$
		$a = 0.2$		
Mean	0.03191	0.02693	0.003537	0.003323
Standard deviation	0.03921	0.03502	0.004200	0.004192
Volatility coefficient	1.28871	1.00712	1.187441	1.261510
		$a = 0.5$		
Mean	0.01521	0.01672	0.001381	0.001404
Standard deviation	0.05902	0.07801	0.005902	0.005898
Volatility coefficient	3.78860	4.65669	4.273692	4.199560
		$a = 1$		
Mean	0.02513	0.01069	0.002877	0.002756
Standard deviation	0.1165	0.1108	0.01179	0.01173
Volatility coefficient	4.2112	10.35642	4.09681	4.25761
		$a = 1.5$		
Mean	0.001744	0.002202	0.001910	0.001775
Standard deviation	0.017100	0.021628	0.017262	0.017180
Volatility coefficient	9.802840	9.821980	9.034320	9.676200
		$a = 2$		
Mean	0.006229	0.003213	0.006427	0.006033
Standard deviation	0.023712	0.022446	0.023920	0.023774
Volatility coefficient	3.806440	6.984651	3.721341	3.940150

Table 1 Basic characteristics of the discussed CPI bias measures for the given values of the parameter a.

We assume that random vectors of prices and quantities are as follows ($P^t = a \cdot P^s$ for some parameter a, Q^t are defined[2] as $\frac{1}{a}Q^s$ and we present below only first five commodities):

$P^s = [U(400,700), U(1000,6000), U(3,9), U(3000,7000), U(100,500),...]'$,

$Q^s = [U(30000,70000), U(100,500), U(300,900), U(20000,50000), U(300,900),...]'$,

where $U(a,b)$ denotes a random variable with the uniform distribution that has values in $[a,b]$ interval. We generate values of these vectors in $n = 100000$ repetitions. After calculations we get the results[3] presented in Table 1.

5. Conclusions

Let us notice that we observe a positive expected value of the commodity substitution. This observation corresponds to the results coming from report of the Boskin Commission, where we can find the estimated value of the commodity substitution bias on the level of 0,004 (Boskin et al. [5]). The similar conclusion can be also found in Cunnigham[4] [7]. We can notice that the estimator of CPI bias calculated as $I_{La} - I_T$ is different in his expected value and has the highest volatility coefficient (other estimators have similar values of this coefficient). Taking into consideration only the expected value of the commodity substitution bias we can find high similarity between the measures based on the Fisher and Białek formulas. However, the scale of the commodity substitution bias does not seem to depend on the parameter a, which describes the changes in prices and quantities.

References

[1] Afriat, S.N., 1972. The Theory of International Comparisons of Real Income and Prices. In: Daly, D.J. (ed.), International Comparisons of Prices and Outputs, New York: Columbia University Press, 13-69.
[2] Białek, J,. 2012a. Propozycja indeksu cen. Wiadomości Statystyczne 7, 13-24.
[3] Białek, J., 2012b. Proposition of a General Formula for Price Indices. Communications in Statistics -Theory and Methods 41(5), 943-952.
[4] Białek, J., 2013. Simulation study of an original price index formula. Communications in Statistics: Simulation and Computation (DOI: 10.1080/03610918.2012.700367).

[2] We assume the classic model when the prices and quantities are negative correlated.
[3] To read more about estimation of mean value and variance and the bias of this estimation see Żądło [19], Gamrot [12], Małecka [14] or Papież, Śmiech [15].
[4] In this raport the commodity substitution bias is in the interval 0 - 0.001.

[5] Boskin, M.J., Dulberger E.R., Gordon R.J., Griliches Z., Jorgenson D., 1996. Toward a More Accurate Measure of the Cost of Living. Final Raport to the Senate Finance Committee from the Advisory Commission to Study the Consumer Price Index.
[6] Clements, K., Izan, H.Y., 1987. The Measurement of Inflation: A Stochastic Approach. Journal of Business and Economic Statistics 5(3), 339-350.
[7] Cunningham, A.W., 1996. Measurement biases in price indexes: an application to the UK's RPI. Bank of England, Working Paper Series 47.
[8] Diewert, W.E., 1974. Functional forms for revenue and factor requirement functions. International Economic Review 15, 119-130.
[9] Diewert, W.E., 1976. Exact and superlative index numbers. Journal of Econometrics 4, 114-145.
[10] Diewert, W.E., 1983. The theory of the cost-of-living index and the measurement of welfare change. Price level measurement: proceedings of a conference sponsored by Statistics Canada, Diewert, W.E., Montmarquette, C. (eds.), Ottawa: Statistics Canada, 163-233.
[11] Diewert, W.E., 1993. The economic theory of index numbers: a survey. Essays in index number theory, vol. 1, Diewert, W.E., Nakamura, A.O. (eds.), Amsterdam, 177-221.
[12] Gramot, W., 2007. Mean Value Estimation Using Two-Phase Samples with Missing Data in Both Phases. Acta Appl Math 96, 215-220.
[13] Hałka, A., Leszczyńska A., 2011. Wady i zalety wskaźnika cen towarów i usług konsumpcyjnych – szacunki obciążenia dla Polski. Gospodarka Narodowa 9, 51-75.
[14] Małecka, M., 2011. Prognozowanie zmienności indeksów giełdowych przy wykorzystaniu modelu klasy GARCH. Ekonomista 6, 843-860.
[15] Papież, M., Śmiech, S., 2013. Causality in mean and variance within the international steam coal market. Energy Economics 36, 594-604.
[16] Pollak, R.A., 1989. The theory of the cost-of-living index. Oxford: Oxford University Press.
[17] Samuelson, P.A., Swamy, S., 1974. Invariant economic index numbers and canonical duality: Survey and synthesis. American Economic Review 64, 566-593.
[18] White, A.G., 1999. Measurement Biases in Consumer Price Indexes. International Statistical Review 67 (3), 301-325.
[19] Żądło, T., 2006. On prediction of total value in incompletely specified domains. Australian & New Zealand Journal of Statistics 48(3), 269-283.
[20] von der Lippe, P., 2012. Some short notes on a price index of Jacek Białek. Ekonometria (Econometrics) 35, 76-83.